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Fostering Geometrical Measurement Skills to Enhance Geometrical Measurement Performance in Primary Schools

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Abstract

Geometrical measurement becomes one of the most critical topics in primary mathematics which require the understanding of the underlying concepts and not just focusing solely on formulas. Unfortunately, the teaching and learning of geometrical measurement is usually based on rules, procedures and formulas. Besides, research have also shown that the scarcity of the conceptual understanding significantly contributes to the poor geometrical measurement understanding. Thus, the focus of this study is to clarify the specific types of geometrical measurement skills that contribute to the geometrical measurement conceptual understanding. The research method was based on literature review search strategy through online database such as Google Scholar and Scopus. The results of this study indicate that in order to grasp the conceptual understanding of geometrical measurement, pupils need to be proficient in geometrical measurement skills. From the literature, it can be concluded that geometrical measurement skills involve the integration of spatial skills and numerical skills. Thus, the findings of this study led us to draw conclusions on the skills that guide primary-level pupils to grasp conceptual understanding in geometrical measurement. Subsequently, the findings may also provide guidance for potential researchers and teachers to evaluate the performance of pupils in geometrical measurement such as developing an alternative assessment tool.

Keywords: Geometrical Measurement, Geometrical Measurement Skills, Numerical Skills, Spatial Skills.

INTRODUCTION

Geometrical measurement has a special and imperative role over almost every mathematics curriculum especially, in primary mathematics. Geometrical measurement can be defined as the measurement of perimeter, area and volume in one, two and three dimensions respectively. A wealth of literature studies have documented that the concept of geometrical measurement plays an essential role not only in mathematics but also forms the basis of other subjects such as physics, biology, chemistry, geology, geography, architecture and art (Crites et al., 2018; Fuson et al., 2010; Kim et al., 2017; Pytlak, 2019). Furthermore, geometrical measurement is seen as an intrinsic part of our daily lives, and we are unaware that it discreetly teaches us the fundamentals of logical thinking and reasoning (Pytlak, 2019). As a result, it is not surprising that geometrical measurement has been taught as early as preschool and has since been developed and expanded throughout primary and secondary schools (Siti Nur Annisa & Lim, 2021).

In view of its crucial position in STEM (Sciences, Technology, Engineering and Mathematics) education and our lives, pupils should fully understand the basic principles of geometrical measurement and not just focus entirely on formulas (Machaba, 2016; Tan Sisman & Aksu, 2016). However, research have shown that pupils have a poor understanding of geometrical measurement due to the scarcity of conceptual understanding (Browning, Edson, Kimani, & Aslan-Tutak, 2014; Kim & Oláh, 2019; Tan Sisman & Aksu, 2016). This can be proved by a study conducted in Slovakia, which found that primary school pupils had trouble understanding geometrical measurement due to their difficulty in visualising figures or objects, which prevents them from understanding and synthesising the figures (Csachová & Jurečková, 2019). Furthermore, Jirotková et al. (2019) discovered that primary school pupils in the Czech Republic and Italy struggled with geometrical measurement because they were confused about the relationship between perimeter and area. This is supported by a research conducted with primary school pupils in Indonesia, in which the pupils reported that relying on complicated formulas makes the topic appear difficult and uninteresting because they are unable to develop and investigate the underlying concepts themselves (Laurens et al., 2018).

The depicted problems occur not just globally, but also locally. Recognizing the significance of geometrical measurement in mathematics, Malaysia assesses this topic in both primary and secondary school examinations, namely the Primary School Achievement Test (UPSR), School- Based Form Three Assessment (PT3) and Malaysia Certificate of Education (SPM). However, according to the Malaysian Examination Syndicate's Answer Quality, pupils continue to struggle with understanding the basic concept of geometrical measurement (Malaysian Examination Syndicate, 2010, 2012, 2013, 2014, 2017). Furthermore, the performance of Malaysian secondary school students in geometrical measurement is concerning, with Malaysia placing below the average level, ranking lower than nations such as Japan, South Korea, the United States, Australia, Thailand, and Singapore in TIMSS 2015 (Chog et al., 2018).The root of the problem is that most of the pupils have learned geometrical measurement by memorizing formulas and applying step-by-step procedures without addressing the underlying concepts of geometrical measurement (Crites et al., 2018; Smith, Males, & Gonulates, 2016). Moreover, past studies have revealed that the problem that leads to a paucity of conceptual understanding of geometrical measurement is because of the absence of assessment of the skills associated to geometrical measurement (Crites et al., 2018; Hannighofer et al., 2011; Vasilyeva et al., 2009). Thus, in responding to the above-mentioned call, this study seeks to clarify the specific types of geometrical measurement skills that are essential for pupils to grasp the conceptual understanding of geometrical measurement.

METHODOLOGY

Data Collection and Analysis

To collect data for this study, an online database technique was used through Google Scholar and Scopus to search for relevant published journal articles and conference papers. Four keywords were employed in the search strategy as follows: geometrical measurement, geometry and measurement, geometry and measurement and spatial skills, geometry and measurement and numerical skills. At the first stage, based on the selected keywords, all types of articles from various fields, such as peer-reviewed journals, were selected, including concept papers, research papers, systematic review papers, meta-analysis papers, and conference papers from January 2013 to August 2021. Then, only research articles and review papers were designated as primary sources at the second stage. At the third stage, the selected articles from stage two were reviewed and qualitatively analyzed, focusing exclusively on fundamental facts of geometrical measurement and skills related with geometrical measurement, particularly spatial and numerical skills. However, due to the study's restricted resources, several resources prior to the year 2013 were used to support the study's findings.

RESULTS

This section will describe the results based on the research questions. The main aim of this study is to clarify the specific types of geometrical measurement skills that contribute to geometrical measurement conceptual understanding. The results of this study were analyzed qualitatively, and geometrical measurement skills were divided into two parts, namely spatial skills and numerical skills. Next, spatial skills were divided into two parts, namely, spatial visualization skills and spatial structuring skills.

Lastly, numerical skills were divided into two parts, namely, counting and computing skills and applying formula skills. Table 1 summarizes the findings in relation to the research question.

Table 1. Summarize of Findings.

Geometrical Measurement Skills			
Spatial Skills		**Numerical Skills**	
Casey et al. (2011), Tůmová & Vondrová (2017), Pavlovičová & Švecová (2015), Ovadiya et al. (2019), Slezáková et al. (2019), Hoth et al. (2019), Hawes et al. (2019), Crites et al. (2018), Okamoto et al. (2014) & Verdine et al. (2016)		Hawes et al. (2017), McCrink & Opfer (2014), Cipora et al. (2018), Casey et al. (2011) & Tůmová & Vondrová (2017), Hawes et al. (2019) & Crites et al. (2018)	
Spatial Visualization Skills	**Spatial Structuring Skills**	**Counting and Computing Skills**	**Applying Formula Skills**
Pavlovičová & Švecová (2015), Ovadiya et al. (2019), Liu et al. (2019),Sáenz-Ludlow & Jiménez (2019), Jirotková et al. (2019), Wahab et al. (2016), Hawes et al. (2019), Crites et al. (2018), Tůmová & Vondrová (2017) & Tan Sisman & Aksu (2016)	Pytlak (2019), Rütten & Weskamp (2019), Jones & Tzekaki (2016), Reinhold et al. (2013), Scandpower & Doorman (2014), Crites et al. (2018), Tůmová & Vondrová (2017), Battista et al. (2017), Tan Sisman & Aksu (2016) & Newcombe et al. (2019)	Achdiyat & Utomo (2018), Casey et al. (2011), Khine (2016), Crites et al. (2018) & Tůmová & Vondrová (2017)	Nurwijayanti et al. (2018), Casey et al. (2011), Crites et al. (2018) & Tůmová & Vondrová (2017)

Geometrical Measurement

Geometrical measurement is a critical field of study, especially in middle grades, i.e., between the ages of 8 and 12 years (Crites et al., 2018). Geometrical measurement refers to the measurements in one, two-, and three-dimensions, including perimeter, area and volume and covers almost all of the essential elements of mathematics (Kim et al., 2017). For this reason, geometrical measurement has already been introduced since the first years of school and often begins with the learning of how to compare measurements between objects, (e.g., shorter than or longer than) before continuing with more complicated geometrical measurement concepts in the following years Outhred, Mitchelmore, McPhail and Gould (2003). Besides, studies including the Trends in International Mathematics and Science Study (TIMSS), have shown that geometrical measurement is one of the core topics in mathematics in which the geometrical measurement content domain for TIMSS in Grade 4 and Grade 8 was 35 percent and 30 percent respectively (Chog, Wong and Halim, 2018). However, geometrical measurement is found to be an area of weakness for primary and also secondary students (Crites et al., 2018; Tůmová & Vondrová, 2017). This is due to the pupils' learning routine which only relies on memorizing formulas and applying step-by-step procedures without actually understand the underlying meaning of the formulas (Crites et al., 2018; Smith, Males, & Gonulates, 2016). One of the reasons for this problem is that students are not exposed to assessments involving the skills needed to grasp the conceptual understanding of geometrical measurement (Crites et al., 2018; Hannighofer et al., 2011; Vasilyeva et al., 2009). In order to resolve this problem, specific mathematical knowledge and skills related to geometrical measurement need to be carefully defined (Clements, Sarama, & Liu, 2008; Tůmová & Vondrová, 2017).

Geometrical Measurement Skills

One of the important skills to ensure that primary-level pupils obtain a clear conceptual understanding of geometrical measurement is geometrical measurement skills (Barrett et al., 2012). At the primary-level, geometrical measurement skills focus mainly on geometrical measurement i.e., perimeter, area and volume measurement (Steele, 2006). However, primary school pupils generally still do not acquire a clear understanding of geometrical measurement because of an inadequacy of geometrical measurement skills (Tůmová & Vondrová, 2017). Therefore, the goal of this study is to define and explain the geometrical measurement skills needed by primary level pupils to grasp the conceptual understanding of geometrical measurement. The findings in Table 1 demonstrate that reviews of the literature establish that geometrical measurement skills are not a unitary construct. Research in the field of measurement has shown that the skills that predicts the level of geometrical measurement skills of primary school pupils are spatial skills and numerical (Casey et al., 2011; Cipora et al., 2018; Crites et al., 2018; Hawes et al., 2017, 2019; Hoth et al., 2019; Ovadiya et al., 2019; Pavlovičová & Švecová, 2015; Slezáková et al., 2019; Tůmová & Vondrová, 2017). Barrett et al. (2012) mentioned in their study that spatial skills and numerical skills are included as geometrical measurement skills, since spatial skills could enable pupils visualise and make sense of objects around them and relate them to numerical skills to better understand the meaning of geometrical measurement formulas. Besides, Hawes et al. (2017) stated in their study that pupils with good spatial skills will certainly perform well in numerical skills. However, there has been little study of the relationship between spatial skills and numerical skills in determining the level of geometrical measurement (Casey et al., 2011). Therefore, this research uses literature as a lens to investigate the integration of spatial skills and numerical skills to predict geometrical measurement skills in primary level.

Spatial Skills

Spatial skills are claimed to be crucial for the achievement in science, technology, engineering, and mathematics (STEM) (Uttal & Cohen, 2012). Indeed, most researchers argued that spatial skills play a major role in determining the mathematical achievement (Battista, 1990; Battista & Clements, 1996; Battista, Wheatley, & Talsma, 1982; Casey, Nuttall, Pezaris, & Benbow, 1995; Joan Moss, Zachary Hawes, Sarah Naqvi, Beverly Caswell, Bruce, 2014; Karaman & Toğrol, 2009; Minna Kyttälä, Lehto, Luit, & Hautamäki, 2003; Mix & Cheng, 2012; Okamoto, Weckbacher, & Hallowell, 2014; Pavlovičová & Švecová, 2015; Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2016; Wai et al., 2009). This is shown by the finding of many overlaps between brain regions used for spatial and mathematical processing in various mathematical fields (Amalric & Dehaene, 2016). In addition, from the review of literatures, spatial skills is the most critical skills required to determine the success in geometry and measurement (Battista, 1990; Battista & Clements, 1996; Battista, Wheatley, & Talsma, 1982; Fuson et al., 2010; Karaman & Toğrol, 2009; Pavlovičová & Švecová, 2015).

Previous studies have proven that spatial skills consist of several sub-skills and can be defined in multiple ways (Crites et al., 2018; Linn & Petersen, 1985). A wealth literature studies have indicated that spatial skills are the combination of several sub-skills such as spatial visualization, spatial perception, spatial orientation, spatial rotations, spatial imagery and many others sub-skills comprising various disciplines such as, engineering, mathematics, art, science and cognitive psychology (Hegarty & Waller, 2005; Linn & Petersen, 1985; Miller & Bertoline, 1991). This implies that the definitions of spatial skills are so broad and common that they do not focus on particular spatial skills concerning specific subjects (Uttal & Cohen, 2012). In order to solve this issue, a number of spatial tasks from primary mathematics curriculum have been studied, especially in geometrical measurement, in order to derive the specific spatial constructs to represent the spatial sub-skills for geometrical measurement skills.

Numerical Skills

Numerical skills are an indispensable skill, not only in everyday life, but also in the pursuit of a good career, which makes it a valuable skill in mathematics (Ball et al., 2005; Hoon et al., 2020). Numerical skills include procedures and concepts that represent and manipulate quantities accurately (Newcombe et al., 2019). Thus, the determination of success in mathematics is measured not only by understanding the mathematical concepts on its own, but also on the efficiency of handling mathematical procedures i.e., acquisition of a strong numerical skills (Tan Şişman & Aksu, 2012). However, studies have shown that there is a scarcity of knowledge and experience to define precisely what early numerical skills are ideally suited to make sure that mathematical knowledge is developed without problems. Besides, Aunio & Räsänen (2016) mentioned in their study that poor performance in numerical skills could limit a child's potential in mathematical success. Therefore, this study not only illustrates and explains the specific spatial skills in geometrical measurement, but also numerical skills that are required in geometrical measurement skills.

Spatial Skills in Geometrical Measurement

Spatial skills are one of the most essential skills to predict the success of primary pupils in geometrical measurement (Hawes et al., 2017). This is because geometrical measurement mainly focuses on visualization, which makes it important for pupils to have a high level of spatial skills Pavlovičová & Švecová (2015). Unfortunately, the integration of spatial skills and geometry is scarcely being introduced and applied, especially in the early stages of education (Clements & Sarama, 2011). Therefore, in order to provide a solid foundation for geometrical measurement, the integration of spatial skills into current curriculum is integral, in particular the development of instruction and the spatial skills instrument (Hawes et al., 2017).

The literature defines spatial skills in multiple ways. The definition varies depending on the type of discipline involved, such as art, technology, psychology, mathematics and engineering education. For example, Miller & Bertoline (1991) define spatial skills as the combination of spatial visualization, spatial orientation, spatial perception, spatial rotations and spatial imagery while Linn & Petersen (1985) argued that spatial skills could be classified into the following three classes, including spatial perception, mental rotation and spatial visualization. On the other hand, Hegarty & Waller (2005) presented spatial skills as the spatial imagery which involve mental imagery. Thus, in this study, a wide variety of tasks from the primary mathematics curriculum, in particular geometrical measurement, have been analyzed in order to determine the spatial skills associated with geometrical measurement skills. The findings from the literature as shown in Table 1 led us to draw conclusions that the construct that capture the most requirements for spatial skills in geometrical measurement content domain are spatial visualization (Battista, 1999a; Ben-Haim et al., 1985; Crites et al., 2018; Hawes et al., 2019; Jirotková et al., 2019; Lehrer, 2003; Ovadiya et al., 2019; Pavlovičová & Švecová, 2015; Revina et al., 2011; Sáenz-Ludlow & Jiménez, 2019; Tan Sisman & Aksu, 2016; Tůmová & Vondrová, 2017; Wahab et al., 2016) and spatial structuring (M. Battista et al., 1998; M. T. Battista, 2003; M. T. Battista et al., 2017; Crites et al., 2018; Jones & Tzekaki, 2016; Lehrer, 2003; Newcombe et al., 2019; Pittalis & Christou, 2010; Pytlak, 2019; Reinhold et al., 2013; Revina et al., 2011; Rütten & Weskamp, 2019; Scandpower & Doorman, 2014; Tan Sisman & Aksu, 2016; Tůmová & Vondrová, 2017).

Spatial Visualization

Past studies demonstrated that spatial visualization significantly contribute to learning of geometry and measurement (Battista, 1990; Clements & Sarama, 2014; Delgado & Prieto, 2004). However, most pupils find it difficult to understand the skills of spatial visualization as they experience difficulties in imagining, manipulating, rotating or ordering objects (Jones & Tzekaki, 2016; Yilmaz, 2009). This is due to inadequate of studies in the field of spatial visualization skills in the specific content domain, particularly for primary school pupils (Ryu et al., 2007). Therefore, relevant spatial visualisation skills in the specific content domain of geometrical measurement have been defined and presented in this study.

The definition of spatial visualisation differs in literature. The spatial visualization was initially defined by Guilford & Lacey (1947) as the ability to manipulate, twist, rotate or invert objects. Later, McGee (1979) suggested that spatial visualization refers to the ability to mentally rotate a depicted object, to mentally reconstruct an object, and to mentally manipulate an object. On the other hand, Clements & Battista (1992) and Hegarty & Waller (2005) described spatial visualization as the ability to visualize the objects movements in two-and three-dimensional spaces. Recent research by Revina et al. (2011) has found that spatial visualization relates to manipulating two and three-dimensional objects. Titus & Horsman (2018)) further emphasized that spatial visualization is not only acquiring the skill to mentally manipulate images, but also the skill to rotate images into a different structure and to imagine visually what is inside a solid object. In this study, the explanation of spatial visualization is driven by the content domain of this study, which is geometrical measurement where only certain elements of spatial visualization are included. Thus, the elements of spatial visualization that are required in geometrical measurement content domain include the visualization of object movements in two- and three-dimensional spaces (Clements & Battista, 1992; Hegarty & Waller, 2005; Pittalis & Christou, 2010) and the mental manipulation of two-dimensional and three-dimensional objects

(McGee, 1979; Pittalis & Christou, 2010; Revina et al., 2011; Titus & Horsman, 2018) including moving, ordering, matching and combining units (Clements & Sarama, 2014).

Spatial Structuring

Previous research has shown that spatial structuring is one of the most critical skills to possess in order to be competent in geometrical measurement (Clements & Sarama, 2014; Tan-Sisman & Aksu, 2012). This is because pupils who can imagine and generate such spatial structuring are able to start formulating and abstracting their enumeration process in terms of formulas (Battista & Clements, 1998). Spatial structuring can be defined as the mental operation of structuring an organization for an object or set of objects in space. According to Battista (2004), spatial structuring involves visualizing and conceptualizing the structure and shape of the object mentally through the identification, interrelation and organization of its components. Specifically, in geometrical measurement, spatial structuring stressed the significance of understanding the units, the relationships between units, and the repeating a composite unit (Battista & Clements, 1996; Schifter & Szymaszek, 2003). Thus, by means of systematic and explicit guidance on spatial structuring, pupils are able to organize their units according to row structures involving a unit of units, followed by row-and-column structures where pupils could flexibly think about unit of units focusing either on rows or columns, and lastly an array structure involving the thinking of coordinated rows and columns which would include the thinking of a unit of units of units (Battista et al., 1998; Clements et al., 2018; Outhred & Mitchelmore, 2000; Sarama & Clements, 2009). Furthermore, spatial structuring can lead to the development of multiplicative thinking (Battista, 1999).

Numerical Skills in Geometrical Measurement

Effective geometrical measurement skills include not only a comprehension of important measurement spatial skills but also numerical skills that involve procedural competence for measurement (Casey et al., 2011). Table 1 shows that previous study has proven that numerical skills are necessary skills involved in geometrical measurement i.e., perimeter, area and volume (Barrett et al., 2006; M. T. Battista, 2003; Cipora et al., 2018; Crites et al., 2018; Hawes et al., 2017, 2019; Huang & Witz, 2012; McCrink & Opfer, 2014; Tůmová & Vondrová, 2017). According to Battista (2003), numerical operations need to be conceptualized in order to link the formula with the object structure. This is because the use of numerical skills such as the application of formulas based on conceptual understanding is an effective method to solve geometrical problems. Casey et al. (2011) suggested in their study that the numerical skills component of geometrical measurement consists of counting and computing skills, including addition, subtraction and multiplication. Accordingly, it can be concluded from the literature that numerical skills that predict performance in geometrical measurement are the integration of counting, computing and applying formula skills to solve formula-based problems using numerical calculations (Achdiyat & Utomo, 2018; B. M. Casey et al., 2011; Crites et al., 2018; Khine, 2016; Nurwijayanti et al., 2018; Tůmová & Vondrová, 2017).

Skills and Sub-Skills of Geometrical Measurement Skills

According to different authors, geometrical measurement skills is composed by several components or skills: spatial and numerical. Besides most of them agree that the two factors of geometrical measurement skills are the spatial skills which are composed by several sub-skills i.e., spatial visualization and spatial structuring, and numerical skills which are composed by several sub-skills such as counting, computing and applying formula skills. Figure 1 below summarized the skills and sub-skills of geometrical measurement skills to measure geometrical measurement.

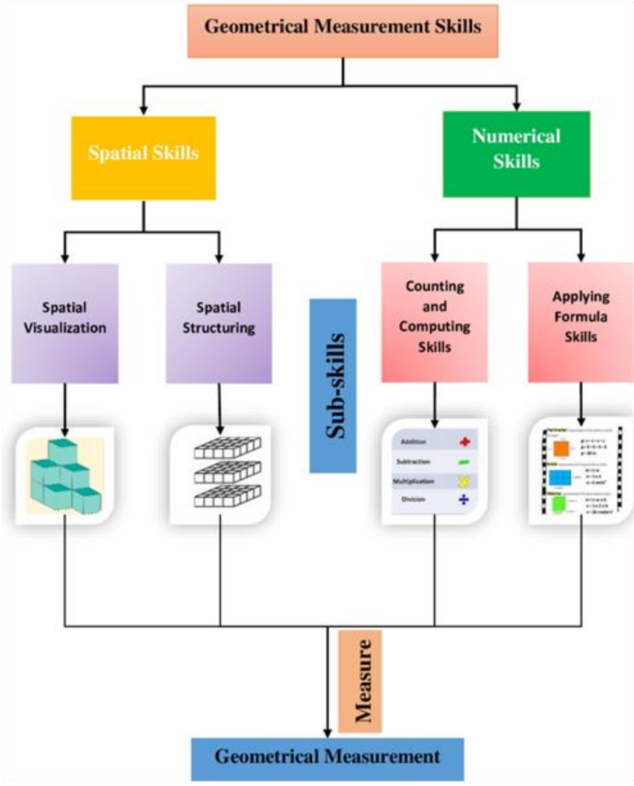

Figure 1. Skills and Sub-Skills of Geometrical Measurement Skills.

Figure 1 shows that in order to grasp the conceptual understanding of geometrical measurement, pupils need to be proficient in geometrical measurement skills. From the literature, it can be concluded that geometrical measurement skills involve the integration of spatial skills and numerical skills. This is due to the fact that spatial skills are crucial skills for pupils to conceptualize the formula for perimeter, area and volume whereas numerical skills are required for pupils to apply the formula correctly. Thus, spatial skills for geometrical measurement can be defined as the integration of spatial visualisation and spatial structuring skills to mentally structure one, two and three-dimensional space from a single unit of measurement to a composite unit and to conceptualize the formula for perimeter, area and volume. Whereas, the subskills for numerical skills for geometrical measurement can be defined as the ability to integrate counting, computing and applying formula skill to solve formula-based problems involving perimeter, area and volume using numerical calculations.

DISCUSSION AND CONCLUSION

The purpose of this study was to define the skills needed to ensure that primary-level pupils obtain a clear conceptual understanding of geometrical measurement. This is because geometrical measurement is considered to be one of the most significant topics in primary mathematics which involve an understanding of the underlying concepts. Thus, the findings of this study led us to draw conclusions on the skills that guide primary-level pupils to understand the underlying concept of geometrical measurement in order to be proficient in geometrical measurement. Subsequently, the findings may also provide guidance for potential researchers and teachers to evaluate the performance of pupils in geometrical measurement such as developing an alternative assessment tool. As a result, the evaluation of strength and weaknesses of primary-level pupils in grasping the underlying concept of geometrical measurement can be identified precisely. Hence, teachers and researchers can strategize systematic teaching approaches, and appropriate remedies to enhance teaching and learning in geometrical measurement.

The development of a new assessment instrument based on the geometrical measurement skills provided in this study will be a strongly recommended study that could be considered in the future. The new alternative assessment tool would provide teachers and pupils with an opportunity to examine and understand the quality of teaching and learning in geometrical measurement that may help guide them through the teaching and learning process. In addition, a potential study on the gender issue will be strongly recommended as it would be possible for the researcher to identify gender differences not only in geometrical measurement skills, but also in numerical and spatial skills. Lastly, future research on the error analysis in geometrical measurement could be carried out to identify the error patterns which could reveal the misconceptions that have been learned in geometrical measurement.

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REFERENCES

Achdiyat, M., & Utomo, R. (2018). Kecerdasan Visual-Spasial, Kemampuan Numerik, dan Prestasi Belajar Matematika. *Formatif: Jurnal Ilmiah Pendidikan MIPA*, *7*(3), 234–245. https://doi.org/10.30998/formatif.v7i3.2234

Amalric, M., & Dehaene, S. (2016). Origins of the brain networks for advanced mathematics in expert mathematicians. *Proceedings of the National Academy of Sciences of the United States of America*, *113*(18), 4909–4917. https://doi.org/10.1073/pnas.1603205113

Aunio, P., & Räsänen, P. (2016). Core numerical skills for learning mathematics in children aged five to eight years – a working model for educators. *European Early Childhood Education Research Journal*, *24*(5), 684–704. https://doi.org/10.1080/1350293X.2014.996424

Ball, D. L., Ferrini-Mundy, J., Kilpatrick, J., Milgram, R. J., Schmid, W., & Schaar, R. (2005). Reaching for common ground in K-12 mathematics education. *Notices of the American Mathematical Society*, *52*(9), 1055–1058.

Barrett, J. E., Clements, D. H., Klanderman, D., Pennisi, S. J., & Polaki, M. V. (2006). Students' coordination of geometric reasoning and measuring strategies on a fixed perimeter task: Developing mathematical understanding of linear measurement. *Journal for Research in Mathematics Education*, *37*(3), 187–221.

Barrett, J. E., Sarama, J., Clements, D. H., Cullen, C., Witkowski-rumsey, C., & Klanderman, D. (2012). Evaluating and Improving a Learning Trajectory for Linear Measurement in Elementary Grades 2 and 3 : A Longitudinal Study Evaluating and Improving a Learning Trajectory 2 and 3 : A Longitudinal Study. *Mathematical Thinking and Learning ISSN:*, *14*(1), 28–54. https://doi.org/10.1080/10986065.2012.625075

Battista, M., Clements, D. H., Arnoff, J., Battista, K., & Borrow, C. V. A. (1998). Students ' Spatial Structuring of 2D Arrays of Squares. *Journal for Research in Mathematics Education*, *29*(5), 503–532. https://doi.org/10.2307/749731

Battista, M. T. (1990). Gender Spatial Visualization Differences in High School Geometry. *Journal for Research in Mathematics Education*, *21*(1), 47–60.

Battista, M. T. (1999a). Fifth graders' enumeration of cubes in 3D arrays: Conceptual progress in an inquiry-based classroom. *Journal for Research in Mathematics Education*, *30*(4), 417.

Battista, M. T. (1999b). The importance of Spatial Structuring in Geometric Reasoning. *Bulletin of the Atomic Scientists*, *32*(10), 34–35. https://doi.org/10.1080/00963402.1976.11455670

Battista, M. T. (2003). Understanding students' thinking about area and volume measurement. In *Learning and teaching measurement: 2003 yearbook* (pp. 3–16). VA: National Council of Teachers of Mathematics.

Battista, M. T. (2004). Applying Cognition-Based Assessment to Elementary School Students' Development of Understanding of Area and Volume Measurement. *Mathematical Thinking and Learning*, *6*(2), 185–204. https://doi.org/10.1207/s15327833mtl0602

Battista, M. T., & Clements, D. H. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. *Journal for Research in Mathematics Education*, *27*(3), 258–292. https://doi.org/10.2307/749365

Battista, M. T., & Clements, D. H. (1998). Finding the Number of Cubes in Rectangular Cube Buildings. *Teaching Children Mathematics*, *4*(5), 258–264.

Battista, M. T., Wheatley, G. H., & Talsma, G. (1982). The Importance of Spatial Visualization and Cognitive Development for Geometry Learning in Preservice Elementary Teachers. *Journal for Research in Mathematics Education*, *13*(5), 332. https://doi.org/10.2307/749007

Battista, M. T., Winer, M. L., & Frazee, L. M. (2017). *How Spatial Reasoning and Numerical Reasoning Are Related in Geometric Measurement*. 355–362.

Ben-Haim, D., Lappan, G., & Houang, R. T. (1985). Visualizing rectangular solids made of small cubes: Analyzing and effecting students' performance. *Educational Studies in Mathematics*, *16*(4), 389–409.

Browning, C., Edson, A. J., Kimani, P. M., & Aslan-Tutak, F. (2014). Mathematical content knowledge for teaching elementary mathematics: A focus on geometry and measurement. *Mathematics Enthusiast, 11*(2), 333–383.

Casey, B. M., Dearing, E., Vasilyeva, M., Ganley, C. M., & Tine, M. (2011). Spatial and Numerical Predictors of Measurement Performance: The Moderating Effects of Community Income and Gender. *Journal of Educational Psychology*, *103*(2), 296–311. https://doi.org/10.1037/a0022516

Casey, M. B., Nuttall, R., Pezaris, E., & Benbow, C. P. (1995). The influence of spatial ability on gender differences in mathematics college entrance test scores across diverse samples. *Developmental Psychology*, *31*(4), 697–705. https://doi.org/10.1037/0012-1649.31.4.697

Chog, S. Y., Wong, T. J., & Halim, A. (2018). Pencapaian Matematik TIMSS 1999 , 2003 , 2007 , 2011 dan 2015 : Di Mana Kedudukan Malaysia Dalam Kalangan Negara Asia Tenggara ? *Malaysian Journal of Higher Order Thinking Skills In Education*, *2*(23), 54–108.

Cipora, K., Schroeder, P. A., & Nuerk, H.-C. (2018). On the Multitude of Mathematics Skills: Spatial-Numerical Associations and Geometry Skill? In *Visualizing Mathematics. Research in Mathematics Education* (pp. 361–370). Springer, Cham. https://doi.org/10.1007/978-3-319-98767-5_18

Clements, D. H., & Battista, M. T. (1992). Geometry and Spatial Reasoning. In *Handbook of research on mathematics teaching and learning (pp.* (pp. 420–464). Macmillan.

Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education*, *14*(2), 133–148. https://doi.org/10.1007/s10857-011-9173-0

Clements, D. H., Sarama, J., Van Dine, D. W., Barrett, J. E., Cullen, C. J., Hudyma, A., Dolgin, R., Cullen, A. L., & Eames, C. L. (2018). Evaluation of three interventions teaching area measurement as spatial structuring to young children. *Journal of Mathematical Behavior*, *50*(July), 23–41. https://doi.org/10.1016/j.jmathb.2017.12.004

Clements, D., & Sarama, J. (2014). *Learning and Teaching Early Math: The Learning Trajectories Approach*. Routledge. https://doi.org/https://doi.org/10.4324/9780203520574

Crites, T., Dougherty, B. J., Slovin, H., & Karp, K. (2018). Geometric Measurement. In *Putting Essential Understanding into Practice: Geometry, 6-8,* (pp. 11–34). National Council of Teachers of Mathematics.

Csachová, L., & Jurečková, M. (2019). Analysis of Problems with Figures in Primary Mathematics. *International Symposium Elementary Mathematics Teaching*, 140.

Delgado, A. R., & Prieto, G. (2004). Cognitive mediators and sex-related differences in mathematics. *Intelligence*, *32*(1), 25–32. https://doi.org/10.1016/S0160-2896(03)00061-8

Fuson, K., Clements, D., & Kazez, S. B. (2010). *Geometry, Spatial Reasoning, and Measurement*. National Council of Teachers of Mathematics, 2010.

Guilford, J. P., & Lacey, J. I. (1947). Printed classification tests (Army Air Forces Aviation Psychology Research Program Report No. 5). In *Washington, DC: US Government Printing Office*.

Hannighofer, J., van den Heuvel-Panhuizen, M., Weirich, S., & Robitzsch, A. (2011). Revealing German primary school students' achievement in measurement. *ZDM - International Journal on Mathematics Education*, *43*(5), 651–665. https://doi.org/10.1007/s11858-011-0357-y

Hawes, Z., Moss, J., Caswell, B., Naqvi, S., & MacKinnon, S. (2017). Enhancing Children's Spatial and Numerical Skills through a Dynamic Spatial Approach to Early Geometry Instruction: Effects of a 32-Week Intervention. *Cognition and Instruction*, *35*(3), 236–264. https://doi.org/10.1080/07370008.2017.1323902

Hawes, Z., Moss, J., Caswell, B., Seo, J., & Ansari, D. (2019). Relations between numerical, spatial, and executive function skills and mathematics achievement: A latent-variable approach. *Cognitive Psychology*, *109*(December 2018), 68–90. https://doi.org/10.1016/j.cogpsych.2018.12.002

Hegarty, M., & Waller, D. A. (2005). Individual Differences in Spatial Abilities. In *The Cambridge handbook of visuospatial thinking* (pp. 121–169). Cambridge University Press. https://doi.org/10.1145/2662253.2662334

Hoon, T. S., Singh, P., Han, C. T., Nasir, N. M., Rasid, N. S. B. M., & Zainal, N. B. (2020). An analysis of knowledge in STEM: Solving algebraic problems. *Asian Journal of University Education*, *16*(2), 131–140. https://doi.org/10.24191/AJUE.V16I2.10304

Hoth, J., Heinze, A., Weiher, D., Ruwisch, S., & Huang, H. (2019). Primary school students' length estimation competence—A cross-country comparison between Taiwan and Germany. *Opportunities in Learning and Teaching Elementary Mathematics*, 201–211.

Huang, H.-M. E., & Witz, K. G. (2012). Children's Conceptions of Area Measurement and Their Strategies for Solving Area Measurement Problems. *Journal of Curriculum and Teaching*, *2*(1), 10–26. https://doi.org/10.5430/jct.v2n1p10

Jirotková, D., Vighi, P., & Zemanová, R. (2019). Misconceptions about the relationship between perimeter and area. *International Symposium Elementary Mathematics Teaching*, 221–231.

Joan Moss, Zachary Hawes, Sarah Naqvi, Beverly Caswell, Catherine D. Bruce, T. F. (2014). Changing Perceptions of Young Children's Geometry and Spatial Reasoning Competencies: Lessons from the "Math for Young Children" (M4yc) Project. *Proceedings of the Joint Meeting 1 - 173 of PME 38 and PME-NA 36*, *1*, 179–181.

Jones, K., & Tzekaki, M. (2016). Research on The Teaching and Learning of Geometry. In *The Second Handbook of Research on the Psychology of Mathematics Education: The Journey Continues* (pp. 109–149).

Karaman, T., & Toğrol, A. Y. (2009). Relationship between gender, spatial visualization, spatial orientation, flexibility of closure abilities and performance related to plane geometry subject among sixth grade students. *Boğaziçi University Journal of Education*, *26*(1), 1–25.

Khine, M. S. (2016). Visual-spatial ability in STEM education: Transforming research into practice. *Visual-Spatial Ability in STEM Education: Transforming Research into Practice*, 1–263. https://doi.org/10.1007/978-3-319-44385-0

Kim, E. M., Haberstroh, J., Peters, S., Howell, H., & Nabors Oláh, L. (2017). A Learning Progression for Geometrical Measurement in One, Two, and Three Dimensions. *ETS Research Report Series*, *2017*(1), 1–26. https://doi.org/10.1002/ets2.12189

Kim, E. M., & Oláh, L. N. (2019). Elementary Students' Understanding of Geometrical Measurement in Three Dimensions. *ETS Research Report Series*, *April*. https://doi.org/10.1002/ets2.12250

Laurens, T., Batlolona, F. A., Batlolona, J. R., & Leasa, M. (2018). How does realistic mathematics education (RME) improve students' mathematics cognitive achievement? *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(2), 569–578. https://doi.org/10.12973/ejmste/76959

Lehrer, R. (2003). Developing understanding of measurement. *A Research Companion to Principles and Standards for School Mathematics*, 179–191. http://ci.nii.ac.jp/naid/10026458755/en/

Linn, M. C., & Petersen, A. C. (1985). Emergence and Characterization of Sex Differences in Spatial Ability : A Emergence and Characterization of Sex Differences in Spatial Ability : A Meta-Analysis. *Child Development*, *56*(6), 1479–1498. https://collab.its.virginia.edu/access/content/group/9181e77b-d425-44e3-9489-db5b45d500a9/Articles - Learning Section/Linn _ Peterson _1985_ Emergence and Characterization of Sex Differences in Spatial Ability.pdf

Liu, R. D., Ding, Y., Gao, B. C., & Zhang, D. (2019). Virtual Manipulatives with Cubes for Supporting The Learning Process. *International Symposium Elementary Mathematics Teaching*, *37*, 427.

Machaba, F. M. (2016). The concepts of area and perimeter : Insights and misconceptions of Grade 10 learners. *Pythagoras - Journal of the Association for Mathematics Education of South Africa*, *37*(1), 1–11. https://doi.org/http://dx.doi. org/10.4102/pythagoras. v37i1.304 Copyright:

Malaysian Examination Syndicate. (2010). *Kupasan Mutu Jawapan UPSR Matematik 015/025/035*.

Malaysian Examination Syndicate. (2012). *Kupasan Mutu Jawapan UPSR Matematik 015/025/035*.

Malaysian Examination Syndicate. (2013). *Kupasan Mutu Jawapan UPSR Matematik 015/025/035*.

Malaysian Examination Syndicate. (2014). *Kupasan Mutu Jawapan UPSR Matematik 015/025/035*.

Malaysian Examination Syndicate. (2017). *Kupasan Mutu Jawapan UPSR Matematik 015/025/035*.

McCrink, K., & Opfer, J. E. (2014). Development of Spatial-Numerical Associations. *Current Directions in Psychological Science*, *23*(6), 439–445. https://doi.org/10.1177/0963721414549751

McGee, M. G. (1979). Human spatial abilities: Psychometric studies and environmental, genetic, hormonal, and neurological influences. In *Psychological Bulletin* (Vol. 86, Issue 5, pp. 889–918). American Psychological Association. https://doi.org/10.1037/0033-2909.86.5.889

Miller, C. L., & Bertoline, G. R. (1991). Spatial visualization research and theories: Their importance in the development of an engineering and technical design graphics curriculum model. *Engineering Design Graphics Journal*, *55*(3), 5–14.

Minna Kyttälä, P. A., Lehto, J., Luit, J. Van, & Hautamäki, J. (2003). Visuospatial working memory and early numeracy. *Educational and Child Psychology*, *20*(3), 65–76.

Mix, K. S., & Cheng, Y. L. (2012). The Relation Between Space and Math. Developmental and Educational Implications. In *Advances in Child Development and Behavior* (Vol. 42). Elsevier Inc. https://doi.org/10.1016/B978-0-12-394388-0.00006-X

Newcombe, N. S., Booth, J. L., & Gunderson, E. A. (2019). Spatial Skills, Reasoning, and Mathematics. *The Cambridge Handbook of Cognition and Education*, 100–123. https://doi.org/10.1017/9781108235631.006

Nurwijayanti, A., Budiyono, & Fitriana, L. (2018). The geometry ability of junior high school students in Karanganyar based on the Hoffer's theory. *Journal of Physics: Conference Series*, *983*(1), 1–7. https://doi.org/10.1088/1742-6596/983/1/012085

Okamoto, Y., Weckbacher, L. M., & Hallowell, D. (2014). How Is Spatial Reasoning Related to Mathematical Thinking and How Important Is Early Exposure to Spatial Activities? *Proceedings of the Joint Meeting 1 - 173 of PME 38 and PME-NA 36*, *1*, 177–209.

Outhred, L. N., & Mitchelmore, M. C. (2000). Young Children ' s Intuitive Understanding of Rectangular Area Measurement. *Journal for Research in Mathematics Education*, *31*(2), 144–167. https://doi.org/10.2307/749749

Ovadiya, T., Fellus, O., & Biton, Y. (2019). Promoting Three-Dimensional Spatial Perceptions of Prisms: The Case of Elementary-School Students Using AR Technology. *International Symposium Elementary Mathematics Teaching*, 288.

Pavlovičová, G., & Švecová, V. (2015). The Development of Spatial Skills through Discovering in the Geometrical Education at Primary School. *Procedia - Social and Behavioral Sciences*, *186*, 990–997. https://doi.org/10.1016/j.sbspro.2015.04.189

Pittalis, M., & Christou, C. (2010). Types of reasoning in 3D geometry thinking and their relation with spatial ability. *Educational Studies in Mathematics*, *75*(2), 191–212. https://doi.org/10.1007/s10649-010-9251-8

Pytlak, M. (2019). Strategies Used by First Grade Primary School Students During the Work on A Geometric Task Concerning Arranging the Plane. *International Symposium Elementary Mathematics Teaching*, 334.

Reinhold, S., Beutler, B., & Merschmeyer-Brüwer, C. (2013). Pre-schoolers count and construct: Spatial structuring and its relation to building strategies in enumeration-construction tasks. *Proceedings of the 37th Conference of the IGPME*, *4*, 81–88.

Revina, S., Zulkardi, Darmawijoyo, & Van Galen, F. (2011). Spatial visualization tasks to support students' spatial structuring in learning volume measurement. *Journal on Mathematics Education*, *2*(2), 127–146. https://doi.org/10.22342/jme.2.2.745.127-146

Rütten, C., & Weskamp, S. (2019). Conjectures and Justifications in Building Towers of Cubes and Cuboids–Design of a Combinatorial Learning Environment. *International Symposium Elementary Mathematics Teaching*, 344.

Ryu, H., Chong, Y., & Song, S. (2007). Mathematically Gifted Students' Spatial Visualization Ability of Solid Figures. *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education*, 137–144.

Sáenz-Ludlow, A., & Jiménez, A. J. (2019). The Influence of Diagrammatic Reasoning on A Teacher's Mathematical Experience. *International Symposium Elementary Mathematics Teaching*, 363.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.

Scandpower, F. van N., & Doorman, M. (2014). Fostering Young Children's Spatial Structuring Ability. *International Electronic Journal of Mathematics Education–IΣJMΣ*, *6*, 27–30.

Schifter, D., & Szymaszek, J. (2003). Structuring a rectangle: Teachers write to learn about their students' thinking. *Learning and Teaching Measurement*, 143–156.

Siti Nur Annisa, M. N., & Lim, H. L. (2021). Development and validation of year five geometrical measurement skills instrument. *International Journal of Evaluation and Research in Education*, *10*(3), 956–965. https://doi.org/10.11591/IJERE.V10I3.21439

Slezáková, J., Vighi, P., & Jirotková, D. (2019). The Role of Metaphor in Geometrical Inquiry – Opportunities Offered by Abstract Art. *International Symposium Elementary Mathematics Teaching*, 408.

Smith, J. P., Males, L. M., & Gonulates, F. (2016). Conceptual Limitations in Curricular Presentations of Area Measurement : One Nation ' s Challenges. *Mathematical Thinking and Learning*, *18*(4), 239–270. https://doi.org/10.1080/10986065.2016.1219930

Steele, M. D. (2006). *Middle grades geometry and measurement: Examining change in knowledge needed for teaching through a practice-based teacher education experience*.

Tan-Sisman, G., & Aksu, M. (2012). The length measurement in the turkish mathematics curriculum: Its potential to contribute to students' learning. *International Journal of Science and Mathematics Education*, *10*(2), 363–385. https://doi.org/10.1007/s10763-011-9304-1

Tan Sisman, G., & Aksu, M. (2016). A Study on Sixth Grade Students' Misconceptions and Errors in Spatial Measurement: Length, Area, and Volume. *International Journal of Science and Mathematics Education*, *14*(7), 1293–1319. https://doi.org/10.1007/s10763-015-9642-5

Tan Şişman, G., & Aksu, M. (2012). Sixth grade students' performance on length, area, and volume measurement. *Education and Science*, *37*(166), 141–154.

Titus, S., & Horsman, E. (2018). Characterizing and Improving Spatial Visualization Skills. *Journal of Geoscience Education*, *57*(4), 242–254. https://doi.org/10.5408/1.3559671

Tůmová, V., & Vondrová, N. (2017). Links between Success in Non-measurement and Calculation Tasks in Area and Volume Measurement and Pupils' Problems. *Scientia in Educatione*, *8*(2), 100–129.

Uttal, D. H., & Cohen, C. A. (2012). Spatial Thinking and STEM Education : When , Why , and How ? In *The Psychology of Learning and Motivation* (Vol. 57). Elsevier Inc. https://doi.org/10.1016/B978-0-12-394293-7.00004-2

Vasilyeva, M., Ludlow, L. H., Casey, B. M., & Onge, C. S. (2009). Examination of the psychometric properties of the measurement skills assessment. *Educational and Psychological Measurement*, *69*(1), 106–130. https://doi.org/10.1177/0013164408318774

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. S. (2016). Links Between Spatial and Mathematical Skills Across the Preschool Years. *Monographs of the Society for Research in Child Development*, *82*(1), 4–184. https://doi.org/10.1109/ciced.2018.8592188

Wahab, R. A., Abdullah, A. H. Bin, Abu, M. S. Bin, Bt Mokhtar, M., & Bt Atan, N. A. (2016). A case study on visual spatial skills and level of geometric thinking in learning 3D geometry among high achievers. *Man in India*, *96*(1–2), 489–499.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial Ability for STEM Domains : Aligning Over 50 Years of Cumulative Psychological Knowledge Solidifies Its Importance. *Journal of Educational Psychology*, *101*(4), 817–835. https://doi.org/10.1037/a0016127

Yilmaz, H. B. (2009). On the development and measurement of spatial ability. *International Electronic Journal of Elementary Education*, *1*(2), 83–96.