Asian Journal of Assessment in Teaching and Learning
Vol 12, Issue 2, 2022 (110-121) /eISSN ***2821-2916***

Geogebra Supported Multiple Representations to Enhance Representational Skills in Calculus

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Received: 24 October 2022; **Accepted:** 16 November 2022; **Published:** 21 November 2022

To cite this article (APA): Nigusse, G. A., & Kassa Michael. (2022). Geogebra Supported Multiple Representations to Enhance Representational Skills in Calculus . *Asian Journal of Assessment in Teaching and Learning*, *12*(2), 110–121. https://doi.org/10.37134/ajatel.vol12.2.10.2022

To link to this article: https://doi.org/10.37134/ajatel.vol12.2.10.2022

Abstract

The advents of multiple interface technologies have made mathematical concepts accessible to students. However, the integration of these technologies into mathematics classroom in Ethiopian universities is at its infant stage. The primary focus of this study was designed to delineate the potential impact of GeoGebra supported multiple representations on students' abilities and levels of representations implementation in calculus learning. Mixed method with pretest and posttest quasi experimental design of non-equivalent groups was implemented. Three intact groups of first year first semester of social science students were formed. The groups were taught with GeoGebra supported multiple representations (MRT), multiple representations (MR) and conventional (CG) approaches. Pretest and posttest on representation implementation were administered. Analysis of Covariance (ANCOVA) and structure of observed learning outcome (SOLO) model were used to compare and level students' score. The ANCOVA result reveals that there was no significant difference among the groups on the adjusted mean of the posttest after controlling the pretest (F (2, 160) = .94, P = .391, Partial η^2 =.012). According to the SOLO model, majority of students of each group was in the multi-structural level (87% of the MRT, 61% of the MR and 70% of the CG).This result informs that students failed to get benefit from the synergetic power of multiple representations. Interview result confirmed that representations implementation is characterized by nature of the problem and solution purpose. It is recommended that further research is required with different participants to generalize to the entire population.

Keywords: Geogebra, Multiple Representations, Representation Implementation, Solo Taxonomy, Deft Framework

INTRODUCTION

Inherently, mathematics is endowed with multiple representations (MRs) used for problem solving, conceptual understanding, reasoning and disciplinary discourse. These representations make abstract mathematical concepts accessible to students. Upon the advent of versatile computer technologies, MRs becomes a conspicuous instructional approach in mathematics to create a cohesive and comprehensive conceptual understanding on students' side. The ability of MRs is the characterization of students who excel in mathematics (Arifah , 2020). The most commonly used representation types in a typical calculus classroom instruction are numerical, graphical, algebraic, verbal, and their possible combinations. The notion of MRs is the delivery of a concept using at least two different representation types (Shaaron Ainsworth, 2006; AN Arifah, 2020). The advantage of learning mathematics with MRs are explained from the perspectives of cognitive maturity (Jerome Seymour Bruner, 1966) , information processing theory (James M Clark & Allan Paivio, 1991) , multimedia learning theory (Shaaron

Ainsworth, 2014) , socio-cultural theory (Vygotsky , 1978) and pedagogical functions (Shaaron Ainsworth, 2006). MRs are used for various reasons. As a result, the assessment of success with MRs depends on the purpose of their implementation in class room instruction. In the process of designing an appropriate multi-representational learning environment to support students' learning, the rhythm must be based on the pedagogical functions that MRs provides along with design parameters and cognitive tasks (Ainsworth, 2006).

The chalk and board approach is no longer supportive for implementing MRs in mathematics classroom. However, the advent of multiple interface computer technologies has increased the quantity and quality of using MRs in mathematics instruction. The emergence of GeoGebra makes MRs more accessibly with more quantities and qualities in calculus learning (AN Arifah, 2020). To get the optimum benefits that MRs can offer, students need to develop multiple representation abilities: the ability of creating, interpreting, implementing and translating multiple representation for problem solving (Nicole L Fonger, 2019). MRs ability exhibits the behavior of students who excel in mathematics. These abilities are equally important for novices and experts (Mathilde Kjær Pedersen, Cecilie Carlsen Bach, Rikke Maagaard Gregersen, Ingi Heinesen Højsted, & Uffe Thomas Jankvist, 2021). However , novices lack the required abilities for MRs (N Nurrahmawati, C Sa'dijah, S Sudirman, & M Muksa, 2019; Dwi Rahmawati, 2019). The ability with MRs enables seamlessly translate one representation to another and flexibly implement various representations in solving mathematical problem. These abilities are at the heart of successful mathematics learning (Kwaku Adu-Gyamfi, Lee V Stiff, & Michael J Bossé, 2012; Michael J Bossé, Kwaku Adu-Gyamfi, & Meredith R Cheetham, 2011; Nicole L Fonger, 2019).

MRs as method of instruction is defined as the implementation of two or more than two different representations simultaneously in classroom instruction when a single representation is no longer adequate to provide a complete picture of a concept (Shaaron Ainsworth, 2006; Nicole L Fonger, 2019; N Nurrahmawati et al., 2019). Representation implementation refers ,in this study, the use of various forms of mathematical representation in solving a problem(Kwaku Adu-Gyamfi et al., 2012; Dona Afriyani, Cholis Sa'dijah, Subanji Subanji, & Makbul Muksar, 2018). The key competency for meaningful learning of mathematics is representational fluency (RF), which refers to the ability to create, interpret, translate between, and connect MRs (Nicole L Fonger, 2019). Representation implementation fluency (RIF) is one dimension of RF that is considered to be a foundation for building up of conceptual understanding and mathematical thinking. Flexibility among multiple mathematical representations is the means through which a student may develop cohesive mathematical processes; know the potential and deficiency of a particular representation in specific situation; and choosing the appropriate representation among the available representations with justifiable reasons (Kirsten Berthold, 2006; Martina A Rau, 2017).

Many researchers reported that students perennially demonstrate difficulty in correctly accomplish the translation task (Kwaku Adu-Gyamfi et al., 2012; Dona Afriyani et al., 2018; Michael J Bossé et al., 2011; Dwi Rahmawati, Subanji Purwantoa, Erry Hidayanto, & Rahmad Bustanul Anwar, 2017). Numerous factors interact to make some translations more difficult than others. A widespread of research results with the courses ranging from algebra to calculus indicate that there is a lost in translation when attempting to go from one representation of a mathematical situation or relationship to another (Adu-Gyamfi et al., 2012). The degree of difficulty is categorized into student-centered factors and content centered factors (Bossé et al., 2011). Student-centered factors are including: the translation action, dual translation (through intermediate translation) and classroom experience. The factors related to the representation type include: different representation types required different interpretation techniques (like local versus global, or syntactic versus semantic), some translation are inherently more complex, and some require greater number of steps to accomplish (Rahmawati et al., 2017). The number of fact gaps associated with either the source or target representation involved in a translation may speak to the difficulty of the translation (Adu-Gyamfi et al., 2012; Bossé et al., 2011).

Representation implementation fluency (RIF) is explained by some level of sophistication ranged from lower level to extended high level of learning outcome (Nicole L Fonger, 2019). RIF is the ability to use various representations with some level of sophistication in solving mathematical problems (Rahmah Johar & Khairiyah Rahma Lubis, 2018). The representation implementation involves explaining mathematical ideas in various forms, translating among mathematical forms and interpreting mathematical phenomenon with various mathematical forms including numerical, graphical, algebraic and verbal. Hence, good problem solvers are sufficiently flexible in the use of a variety of different representations. The leveling of students' sophistication in RIF was based on Structure of the Observed Learning Outcome (SOLO) taxonomy (Afriyani et al., 2018).

Now a days, many challenges are reported on the implementation of MRs in mathematics instruction (Shaaron Ainsworth, 2006; AF Samsuddin & H Retnawati, 2018). In the availability of MRs implementation, it is a fact that there is an urge of preferring , translating, implementing and interpreting of representations (Shaaron Ainsworth, 2006). However , students often fail to do so due to lack of RF (Dwi Rahmawati, 2019). Students also lack to relate the representation with the underline mathematical concept (AF Samsuddin & H Retnawati, 2018). Nicole L Fonger, Jon D Davis, and Mary Lou Rohwer (2018), in turn, reported that technology supported instruction enabled student to enhance representation implementation fluency. Students have a huge gap in flexible implementation of MRs during problem solving (Kwaku Adu-Gyamfi, Michael J Bossé, & Kayla Chandler, 2015; Kwaku Adu-Gyamfi et al., 2012; Michael J Bossé et al., 2011). Peggy Van Meter, Alexandra List, Doug Lombardi, and Panayiota Kendeou (2020) indicate that novice (students) fundamentally lack RF that enable them to understand the underline concepts. This study was framed to investigate the effect of GeoGebra supported multiple representations approach on students' representation implementation fluency in learning calculus. Therefore, this study seeks to answer the following research questions:

1. What is the effect of GeoGebra supported multiple representations approach on students' performance in representation implementation of calculus problems?
2. What is the level of students' sophistication of representation implementation based on the SOLO taxonomy across groups in calculus learning?
3. How do students prefer the representation type that they use to solve calculus problems?

THEORETICAL ORIENTATION

Multiple representations approach is conceptualized, in this study, revisiting the same mathematical concept using verbal, numerical, algebraic, and graphical with the possible combination for building up levels of sophistication for representations implementation with the support of GeoGebra. Implementing multiple representations for solving mathematical problems has a clear level of sophistication and pedagogical functions. The SOLO taxonomy was used to assess students' levels of sophistication in multiple representation implementation (Biggs and Collis , 1982) ranged from surface -to - deep – to conceptual understanding. The taxonomy is used to create the characteristics of students' mathematical understanding in solving multiple representation tasks (Fonger, 2019). The taxonomy has five levels: pre-structural (no understanding), uni-structural (knowing single aspect), multi-structural (understanding various aspect of the concept), relational (relating the aspects to form a structure) and abstract extended (transferring to other concepts) (Dona Afriyani et al., 2018). The DeFT (Design , Functions, Task) framework (Shaaron Ainsworth, 2006) was used to determine the pedagogical functions of multiple representations implementation in the teaching and learning process. The DeFT framework considers many dimensions of the multiple representations to ensure the real influence the synergy of multiple representations. The dimensions considered in DeFT are the design parameters, the different pedagogical functions, and the cognitive tasks of MRs (Shaaron Ainsworth, 2006). The three pedagogical functions of MRs include: complement (providing additional information), constrain (avoiding possible mis-interpretation) and construct (construct deeper understanding (Shaaron Ainsworth, 1999).

RESEARCH METHODOLOGY

Research Design

The overall aim of this study was to explain students' sophistication of representation fluency in solving differential calculus problems as a result of GeoGebra supported multiple representations instructional approach. This paper is excerpt of a PhD dissertation paper whose data was mined from the full scale research work. The study was implemented multi-treatment pretest and post-test non-equivalent group quasi-experimental research design on purposefully selected universities and randomly assigned three groups of first year social science students. The study was intended to compare the effects of three differentiated approaches: GeoGebra supported multiple representations approach (MRT), multiple representations approach (MR) only and that of comparison group with traditional instruction (CG) on students' representation implementation skills. The study was conducted in 2019/20 on first year during the first semester, and the students were from the social science stream, who enrolled for the course mathematics for social science, at Jigjiga University (JJU) and Kebri-Dehar University (KDU). The experimental and comparison groups in the quantitative study were assigned at random to explore the effects of three differentiated approaches: GeoGebra supported multiple representations approach (labeled as MRT), multiple representations approach (labeled as MR) and the conventional approach (labeled as CG) on students' performance in representation implementation on differential calculus concepts. As a result, the study's research design could be summarized as presented in Table 1.

Table 1. The diagrammatic representations of nonequivalent comparison group research design

Groups	Pre-test	Treatments	Post-test
Experimental group one	O_1	E_1	O_2
Experimental group two	O_1	E_2	O_2
Comparison group	O_1	X	O_2

Where: $\mathbf{O_1}$ is pre-test for the experimental and comparison groups

$\mathbf{O_2}$ is post-test for experimental and comparison groups

$\mathbf{E_1}$ is treatment for experimental group1 (received GeoGebra supported multiple representations approach)

$\mathbf{E_2}$ is treatment for experimental group2 (received multiple representations approach alone)

X is treatment for comparison group (received the actual existing instruction)

Similarly, qualitative data were collected to investigate the students' sophistication on representation implementation in solving calculus problems was leveled using the SOLO model.

Sample and Sampling Technique

Jigjig University (JJU) and Kebri-Dehar University (KDU) were purposely selected due to the parallel admitting of first year students in the 2019/20 academic year, the coinciding of time with the approval of proposal, similarity of the universities in terms of geographical and cultural context. One intact class from JJU and two intact classes from KDU were randomly selected as participants of this study. The intact class from JJU was assigned into the GeoGebra supported multiple representations approach (labeled as MRT) and the two intact classes from KDU were assigned into multiple representations approach (labeled as MR) and comparison group (labeled as CG). The three groups were selected from first year social science stream students who were admitted based on the newly endorsed higher education educational roadmap and registered for the course mathematics for social science.

Variables of the Study

Intervention groups consisting of three labels (GeoGebra supported multiple representations approach, multiple representations approach alone, and comparison group) were the study's independent variables. Students' representational skills were the study's dependent variables.

Data Collecting Instruments

The data collection instruments of this study were representation implementation fluency test (pretest and posttest) and interview. The problems in the test were posed by different representations and the students were required to solve them using various representations that were suitable for them. Tasks that involve representation implementation could lend themselves to rubric assessment and to other assessment types suitable for open-ended activities. Hence, the students' score on the representation implementation was quantified using rubric assessment technique and leveled their performance using the SOLO taxonomy as illustrated in Table 2.

Table 2. Representation implementation posttest items and expected elucidation

Item	Representation type(s) of the problem	Demonstrating Sophistication in Representation Implementation Fluency.
1	Algebraic	Told to implement for determining limit of a function at a specific number using the combinations of numerical values, graphical illustration, algebraic computation and verbal explanation as accurate as it can.
2	Algebraic	Told to implement for determining limit of a function fail to exist at a specific number using the combinations of numerical values, graphical illustration, algebraic computation and verbal explanation as accurate as it can.
3	Algebraic	Told to implement verbal method in supporting with the other types of representation in the argument for solution
4	Numerical	To be able to estimate where the given function is increasing, decreasing, and has local extreme using multiple representations.
5 (a)-(c)	Numerical	Data of distances covered by a runner was recorded numerically. Be able to estimate when runner is moving toward the motion detector; moving away from the motion detector; give an interpretation of any local extreme values in terms of this problem situation
6	Verbal	To be able to use the concept of the derivative to define what it might mean for two parabolas to be parallel. Construct equation for the two such parallel parabolas and graph them. Are the parabolas everywhere equidistant, and if so, in what sense?
7	Numerical	To be able to produce a derivative table of values from a table of values
8	Numerical	Told to implement algebraic method in association with the other types of representation in the argument for solution
9	Combination algebraic and numerical	Told to implement appropriate representation effectively and accurately and coordinating these representations
10	Graphical	Told to solve the problem using algebraic method in association other types of representations effectively and accurately

Validity and Reliability of the Instruments

With the intention of obtaining reliable and valid information from the data collection instruments, several efforts had been made. The main types of validity that were tried to establish through different mechanisms included: face validity, content validity, construct validity and criterion validity. In order to establish validity of the representation implementation problem, the supervisors' comments were used and the items were modified accordingly. In addition to the supervisors' comments, the opinions of mathematics experts, who are member of the academic staff in mathematics department at JJU, were consulted for checking the validity of concept and appearance from the aspects that it aimed to measure. Regarding to the face validity, the assessors evaluated the appearance of the items in each of the constructs in terms of feasibility, readability, consistency of style and formatting, and the clarity of the language used to the level of the participants experience. Panel of experts were also involved to evaluate content validity of the constructs in the way to ensure each of the constructs incorporates all the items that were essential in which they eliminated irrelevant items in any of the constructs. In addition to the panel of experts, the literature review was used to establish content validities of the constructs.

To establish the reliability of the instruments of each construct, a pilot test was conducted on second year mathematics department students. Thirty students (15 students for the pretest and 15 students for post-test) were participated in the pilot test. The students' solutions on the constructs were assessed through rubrics. Two iterators were involved in assessing the students' work using the predetermined rubrics for scoring students' solution of the items in each of the constructs. As a result of this, student's solution were analyzed separately by two mathematics department academic staff members at JJU and the calculation of the reliability were computed manually using the formula "Consensus/ (Consensus + Dissensus) X 100" recommended by Miles & Huberman (1994), cited in Dwi Rahmawati et al. (2017). The reliability results were obtained to be .81 and .68 for the pretest posttest, respectively, which was moderately reliable (Cohen, 1992) as cited in (Mary L McHugh, 2012).

Treatment Procedure

The MR group was received multiple representations approach, with special focus on the verbal, numerical, graphical, algebraic representations and their possible combinations. In the MRT, some classroom arrangement and classroom shifting was implemented during the intervention. Even if all of the classrooms in JJU have access of electricity infrastructures, there was erratic power supply. These classrooms are not confortable to use educational technologies, since they are designed for chalk and board teaching approach and they have stretched rectangular shape. In the MRT classroom had two phases. First, GeoGebra was used side to side with the chalk and board/ pen and paper in the hall using laptop and liquid crystal display (LCD) technologies. Second, a computer lap was used to practice the students on the GeoGebra worksheets and to construct their own at fly. The time allocation for these sessions of the group was based on the 3 credit hours and 2 tutorial hours of the course. The students were not constrained to use the software only in the class time, but many of the students downloaded and installed in their private electronic devices. They were using it in their dormitory for practicing, experimenting and learning mathematics contents with their own peace.

Due to the fact that the course instructor and the students were novices for the technology, most of the time , online sources were using for the classroom presentation and demonstration as well as for the computer lab practicing and experimentations. In the MRT group, by means of GeoGebra, the teaching and learning of calculus was shifted into more active, where students explored calculus concepts with linked multi-representations, which is often difficult using chalk and board. The CG was taught based on the conventional approach, which was more dominantly algebraic representation. The intervention was lasted for about six weeks.

Methods of Data Analysis

To compare the three groups with regards to their performance on the representation implementation and representation translation in solving calculus problems, appropriate inferential statistics were used depending on the underlined statistical assumptions with regards to the collected data. One way ANOVA was employed to detect variations of the three groups on their RIF pretest. One way ANCOVA was implemented to compare the three groups based on their score on the RIF's posttest using the RIF pretest as covariates. Students' status and sophistication in representations implementation was leveled according to the SOLO model. A narrative analysis was used for the qualitative data to determine students' ways of choosing type.

FINDINGS AND DISCUSSION

Pre-intervention Findings

The pretest results were the base line test scores taken from the three groups just before the intervention had been begun. The data were obtained from two main constructs: representation implementation fluency (RIF) and interview. From the RIF pretest, ordinal data were driven based on the SOLO taxonomy to level students' performance on the representation implementation. Hence, the test results involved both continuous and categorical data. For the continuous data, appropriate inferential statistics were used and for the categorical data, frequency and percentage were applied to describe the status of students before the intervention as shown in Table 3.

Table 3. Means and Standard Deviations of the RIF and TRF pretests for Groups

		RIF pretest	
Group	**N**	***M***	***SD***
MRT	53	20.66	3.45
MR	57	17.82	5.35
CG	54	20.50	3.87

Mean and standard deviations were performed to summarize and describe the data set obtained from the continuous components of RIF pretest. However, this result did not enable to generalize to the target population with certain level of confidence and significance. Hence, generalizing the results to the target population was detained until the appropriate inferential statistics had been performed as shown in Table 4.

Table 4. One-way ANOVA Results for the RIF pretest

Variable	Source	*SS*	*df*	*MS*	*F*	*P*
RIMF Pretest	Between groups	282.93	2	141.46	7.55	.001
	Within Groups	3015.63	161	18.73		
	Total	3298.56	163			

The between subjects one way ANOVA was conducted to compare the three groups on the RIF pretest. The stringent statistical assumptions for the ANOVA model had been checked before it was used for comparing the groups. These assumptions were met. The result reported in table 4 shows that the three groups had statistically significant mean differences on the RIF pretest (F (2,161) = 7.55, P = .001) before the intervention had been begun. This result manifested that students in the three groups had various experiences on the RIF in learning on the pre-calculus concepts before the intervention. The post hoc comparison using the Tukey HSD test verified that the mean scores of the MR (M = 17.82, SD = 5.35) was significantly less than the MRT (M = 20.66, SD = 3.45) and CG (M = 20.50, SD = 3.87) on the RIF pretest as presented in Table 5. The MRT did not differ significantly from the CG on the RIF pretest (P = .972). The base line variations of the groups on the RIF before the intervention was considered as a covariate for the posttest and controlled statistically using ANCOVA.

Table 5. Frequency and Percentage of the SOLO Taxonomy in RIF Pretest with in Group

	SOLO Taxonomy									
	Prestructured		Unistructured		Multistructured		Relational		Extended Abstract	
Group	*f*	*%*	*F*	*%*	*F*	*%*	*F*	*%*	*F*	*%*
MRT	9	17	44	83	1	2	-	-	-	-
MR	24	42	33	58	-	-	-	-	-	-
CG	9	17	44	82	-	-	-	-	-	-

The structure of observed learning outcome (SOLO) taxonomy (Biggs & Collis, 1982) was adapted to identify students' level of sophistication in representation implementation. As it can be observed in table 5, most of the students were in the level of Unistructural (83% of the MRT, 58% of the MR & 82% of the CG). The remaining students were at the levels of Prestructural (17% of the MRT, 42% of the MR & 17% of the CG). These results revealed that the students were at the lower level with respect to the SOLO model in their level of sophistication in representation implementation.

Post-Intervention Findings

Once the researcher become sure that there were no silly errors in the data set collected from the post administrated instruments (for instance, avoiding out of range scores in any of the instruments), a preliminary data analysis was done on the constructs of RIF to check the stringent statistical assumptions for the underlined statistical methods. These preliminary activities paved the way to choose correct and appropriate inferential statistical tools and techniques to address the formulated research questions of this study. As this research design was a non-equivalent groups quasi-experimental research design, most of the statistical tools that were chosen focused on estimating the differences between groups on data scores from the post administrated instruments as a result of the differentiated treatment type. In choosing the right statistic, a number of decisive factors were taken into great considerations, including: type of research question wished to be addressed, the type of items and scales (level of measurement) that constituted the data collection instrument, the nature and characteristics of the available data set and the assumptions for the specified statistical technique (Pallant, 2005).

Based on the descriptive statistics results reported in Table 6, slight variations were detected among the groups on the indicated variable. It would be a hasty generalization ahead of implementing the appropriate inferential statistic, however to provide conclusive information based on the results of the descriptive statistics.

Table 6. Means and Standard Deviations of the RIF posttest

Group	N	RIF Posttest	
		M	***SD***
MRT	53	33.89	4.34
MR	57	32.38	4.37
CG	54	33.69	5.05

A one way ANCOVA was carried out to compare the groups on the RIF posttest while controlling the RIF pretest. Preliminary analyses were carried out to check the statistical assumption for the underlined statistical method and assumptions were met (Julie Pallant, 2013). The result reported in Table 7 shows that there was no statistically significant difference among the groups on the adjusted mean of the RIF posttest after controlling RIF pretest (F (2, 160) = .94, P = .391, $Partial\ \eta^2$ =.012). 1.2 % of the variance on the RIF posttest was explained by the treatment type. This effect size is small range.

Table 7. One Way ANCOVA Results of the RIF posttest using the RIF pretest as Covariate

DV	Source	Type III SS	*Df*	*MS*	*F*	*P*	*Partial* η^2
RIF posttest	RIF pretest	8.80	1	8.80	.43	.515	.003
	Group	38.99	2	19.49	.94	.391	.012
	Error	3304.49	160	20.65			

DV: Dependent Variable
. R Squared = .012 (Adjusted R Squared = -.006) for the RIF posttest

The SOLO taxonomy (JB Biggs & KF Collis, 1982) was used to level students based on the RIF posttest result. As it can be noticed in Table 8, majority of students in each group were situated at the multi-structural level (87% of the MRT, 61% of the MR & 70% of the CG). The remaining number of students located at the uni-structural level (8% of the MRT, 33% of the MR & 30% of the CG). Negligible number of students (4% of the MRT and 5% of the MR) advanced into the levels of Relational and Extended Abstract. Hence, the students' sophistication in representation implementation in solving multiple representations based task was characterized as incomplete comprehension and compartmentalized. These outcomes revealed that 91% of the MRT, 67% of the MR and 70% of the CG were advanced to the higher levels of the SOLO taxonomy after the interventions. With respect the three groups, considerable percentage of students from the MRT was advanced into the higher levels of sophistication in the RIF due to the intervention. These results revealed that the three groups demonstrated a great variation on the sophistication in the representation implementation fluency, in which more students from the MRT group advanced to the higher level after the intervention than students from the other two groups.

Table 8. Leveling of Students on RIF Posttest within Group

	Prestructured		Unistructured		Multistructured		Relational		Extended Abstract	
	F	*%*	*F*	*%*	*F*	*%*	*f*	*%*	*f*	*%*
MRT	-	-	4	8	46	87	2	4	1	2
MR	-	-	19	33	35	61	3	5	-	-
CG	-	-	16	30	38	70	-	-	-	-

Qualitative Data Findings

The purpose of the qualitative data was to explore how the students prefer the representation type that they used to solve calculus problems. Structure interview was used. Six students were involved in the interview of whom three came from the MRT and the other three came from the MR. Different students put different reason for choosing a particular representation type. Based on the students' response, it was observed that they did not stick to a specific representation type, but they recognized each representation type was useful in its own context. Students choose numerical representation for accuracy.

> "Using table enables me to get the exact value better than the others. Using graphical representation, we can analyses global behavior of the function but not the specific value. The algebraic expression is advantageous for solving the problem step by step. When we take verbal expression it shows the condition of change of functional value but does not show at which point the function changes faster or slower (S1MRT). The idea obtained from the MR group stated as a table value is used to obtain specific value of a function find the value of a function at a specific point, but other expressions are difficult to analyze or get the exact value at distinct point (S1MR)."

These students share the view that a table of values gives exact answer but may skip the value of interest, graphs often do not provide a precise value; and working with equations is important for procedural purpose with most susceptible for error. On the other hand, students' implementation of graphical representation depends on the worth of information because it provides global information about the behavior of a given function.

> "I choose graphical representation in solving calculus problems because from graph we can get sufficient information about the behavior of a function on its entire domain (S2MRT). The graph illustrates that the rate of change of a function at all intervals of its domain. Using graph, it can be easily visualize the amount of rate of change of a function (S2MR). By using the graph we can identify the change of height, the graph shows the change of height at which interval of time it grows faster and at which interval of time the height grows quickly (S3MRT)."

Based on these sample students' implementation of the graphical representation in solving calculus problems, it can be confirmed that graph provides sufficient information about the behavior of the function through its entire domain. Students choose algebraic representation based on its procedural purpose.

> "I would use algebraic formula because I can apply all kind of arithmetic operations (S3MRT). I implement formula because it is ease to substitution (S3MR). To find the instantaneous value, we can substitute any value directly into the equation and get the required value easily. The graph is difficult to analyze the exact value. The table gives the exact but we can't know how we get this value, when we see the verbal it is not insightful to find value. It tells the form of the function but does not give the form of the function and the exact value (S2MRT)."

These responses assured that students implement algebraic representation while solving a specific calculus problem was due to its ease of substitution and for procedural purpose. They were confortable to solve the problem by substituting to the explicitly formulated algebraic formula using the algebraic rules and procedures.

The students' responses indicated that verbal representation is chosen because it has *expressing power*.

> "It is unnecessary to do more calculations and take time because the verbal explanation provides the contextual meaning of the problem (S1MRT).Verbal explanation is appropriate than others because it shows the behavior of the function better than others. The equation did not show the limit value and the table value did not show the value of a function (S1MR)."

Verbal expression has the power to contextualize the concept and is used to represent ideas and relations inside and outside the domains of math and science. It is also very expressive, and facilitates ease of communication mathematical ideas with other. Verbal explanation has the potential to contextualize the abstract problem in which the others can reduce and simplify to represent.

Students can also implement the combination of various representations while solving a particular calculus problem. One clear idea obtained from the students' response was that *complementary* nature of two or more representations. The excerpt below indicates this.

> "Since the graph is sketched according to the value of the table both of them give clues to predict the time at which the ball reaches to the ground (S3MRT)."

According to these results, students' representation implementation depends on the purpose and nature of the calculus problem. They did not relay on one representation type that works for all problem types. So, there was no particular representation type that fit for all. The students in the MRT groups used GeoGebra installed in their smart phones to solve a problem using graphical representation. Without any doubt, students in the MRT and MR groups attempted to use MRs in learning calculus concepts. These students indicated that they use multiple understanding of calculus concepts. For example, better understanding the concepts of calculus is associated with their use of MRs

Discussion

Over the past two decades, several efforts had been made to reform calculus curriculum to incorporate multiple representations. Multiple representations as method of instruction is highly demanded in the reformed calculus textbook (Briana L Chang, Jennifer G Cromley, & Nhi Tran, 2016). As a result, students' success in calculus learning is strongly associated with multiple representations abilities. One of these abilities is representational fluency (RF). Representations implementation fluency (RIF) is one of the main components of RF that characterizes students who excel in mathematics. Due to the advent of multiple interface mathematical technologies, quantity and quality of MRs used in a classroom instruction is increased. GeoGebra is one of the dynamic software that assimilates different learning styles using MRs. Hence, this study was mainly focused on the effect of multiple representations approach on students' RIF in learning calculus. .

The concepts of calculus covered in this study include: limits, continuity, derivative and application of derivative of function of single variable. A pretest was administrated to check the equivalence of the three groups on their RIF just before the intervention had been begun. The contents of the pretest were compiled from function concepts, which are a pre-requisite for calculus. The treatment lasted for about six weeks. Upon the accomplishment of the treatment, a posttest was administrated immediately in the beginning of the seventh week of the treatment. The contents of the posttest were compiled from the contents covered during the intervention but with similar constructs with the pretest. The students in the three groups were also categorized into five levels based on their RIF using the SOLO taxonomy.

The three groups were compared on the RIF posttest using the RIF pretest as covariate. The one way ANCOVA results revealed that there was no statistically significant difference among the groups on the adjusted mean of the RIF posttest after controlling the RIF pretest (F (2, 160) = .94, P = .391, $Partial\ \eta^2$ =.012). The manifestation obtained from the effect size that 1.2% of the variances on the RIF posttest, res was explained by the treatment type.

Students' sophistication on the RIF was also analyzed using the SOLO model. In light of this model, students' RIF was portrayed with respect to students' use of one or more than one representations in doing mathematical tasks. Based on the SOLO taxonomy outputs, a huge number of students of each group (86.8% of the MRT, 64.4% of the MR and 70.4% of the CG) reached in the Multistructed level next to the unistructural level (7.5% of the MRT, 33.3% of the MR and 29.6% of the CG). Negligible number of students (3.8% of the MRT and 5.3% of the MR) advanced into the levels of Relational and Extended Abstract. These outcomes revealed that 90.65% of the MRT, 66.7% of the MR and 70.4% of the CG were advanced to the higher levels of the SOLO taxonomy after the interventions. With respect to the three groups, considerable percentage of students from the MRT was advanced into the higher levels of sophistication in the RIF based on the SOLO taxonomy. This result coincides with the finding of Nicole L Fonger (2019) that computer algebra system (CAS) enabled students to upgrade from lesser meaningfulness in representational fluency to more sophisticated reasoning.

Based on the interview result, students' ways of choosing representation type for solving a particular calculus problem depends on the representation type and nature of the problem. The categories emerged from the interview results, included: Students use numerical representation when a sought *exact value* for a problem. They used graphical representation to solve a problem when they required *sufficient information*. According to the students' response, algebraic representation is used to solve a calculus problem by substitution. That is , students used algebraic representation for procedural purpose. The verbal representation has the *expressing power* of a problem from its context without reducing its dimensions. The combination of representations provides complementary information for the students. In line to these results, Meltzer (2005) found that students' response to a problem varies according to the representation type. These results revealed that students contextualized their representation preference based on the nature of the problem and the appropriateness of the representation type.

CONCLUSION

The GeoGebra supported multiple representations approach was more supportive for developing students' level of sophistication in representation implementation, but no variation on representation implementation ability. Moreover, majority of students in the MRT promoted to the higher level of the SOLO model in RIF. Despite the variation in prevalence, the pattern of students' sophistication in RIF was the same. Majority of students were in the multi-structural level. Students' way of choosing representation type depends on the problem type and purpose of its solution. Hence, each representation type has its own usage for solving a particular problem.

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