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Visualization of Intuitionistic Fuzzy B-Spline Space Curve and Its Properties

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Abstract

In this paper, an intuitionistic fuzzy B-spline space curve is defined and some of its properties is introduced. Firstly, intuitionistic fuzzy control point is defined based on intuitionistic fuzzy and geometrical modeling concepts. Each of the control point relation that consists of three function is find and shown. Later, the control point is blended with B-spline basis function and intuitionistic fuzzy B-spline curve is visualized. Some of the control point and space curve properties in the Euclidean space is also discussed throughout this paper.

Keywords Intuitionistic fuzzy B-spline, intuitionistic fuzzy control point, intuitionistic fuzzy set, space curve, control point relation

INTRODUCTION

Curve is an important tools used to visualize collected data or information provided. It is a necessary and inevitable in order to represent data point [1]. The data need to visualize as it can show the trend and nature of the data studied. A curve can be visualized depends on the conditions of the data obtained. The curve became harder and challenging to visualize when there exist uncertainty features in the data. In normal situation, an uncertainty data is remove from a set of data regardless of its effect on the resulting curve. Therefore, the evaluation and analyzing process from the data visualized will be incomplete. The data set should be filtered if there exist an element of uncertainty so that the data can be used to generate curves or surfaces of a model that want to be investigated.

To overcome this matter, intuitionistic fuzzy set (IFS) is used. IFS is a generalization of fuzzy set theory from [2] and was introduced by in [3][4]. The set consists of three component namely degree of membership, non-membership and uncertainty (non-determinacy). Intuitionistic fuzzy set (IFS) have been studied and applied in different fields of science, mathematics, engineering and much more such as in [5][6]. IFS is generally defined by three functions consists of membership, non-membership and uncertainty with the constraint that the sum of these three functions must be equal to one [7].

Research of IFS related to curve and surface have been done by Zulkifly & Wahab. In [8], they introduced an idea of IFS in spline curve and surface which focused on Bézier spline where the curve and surface are blended with intuitionistic fuzzy control point. Wahab et. al [9] discussed intuitionistic fuzzy Bézier model and generated intuitionistic fuzzy Bézier curve using interpolation method. They visualized intuitionistic fuzzy Bézier curve that consists of membership, non-membership and uncertainty curve by blending the Bernstein polynomial with intuitionistic fuzzy control point that have been defined. Later, Zulkifly & Wahab defined intuitionistic fuzzy control point relation (IFCPR) through intuitionistic fuzzy concept with some properties. They illustrate intuitionistic fuzzy bicubic Bézier surface through the approximation method by using data point with intuitionistic features [10]. By using IFCPR, they also generate cubic Bézier curve through interpolation method and intuitionistic fuzzy B-spline curve (IFB-SC) using approximation method [12][13].

The aim of this paper is to generate and illustralized intuitionistic fuzzy B-spline space curve (IFB-SSC) for 2D and 3D universes through interpolation method by using intuitionistic fuzzy control point relation that had been introduced. This paper is organized as follows. Section 1 discussed introduction of

this research. In section 2, some previous works is shown. Section 3 introduces definition of B-spline space and some of its properties. Finally, section 4 will conclude this research.

PRELIMINARIES

In this section, some basic definition of intuitionistic fuzzy point relation that had been introduced in previous research discussed.

Definition 1. [11] Let V,W be a collection of universal space of points with non-empty set, then IFPR is defined as

$$T^{*}=\left\{\left\langle\left(v_i,w_j\right),\mu_T\left(v_i,w_j\right),\nu_T\left(v_i,w_j\right),\pi_T\left(v_i,w_j\right)\right\rangle \mid \left(\mu_T\left(v_i,w_j\right),\nu_T\left(v_i,w_j\right),\pi_T\left(v_i,w_j\right)\right)\in I\right\} \tag{1}$$

where $\left(v_i,w_j\right)$ is an ordered pair of points or coordinate and $\left(v_i,w_j\right)\in V\times W$. $\mu_T\left(v_i,w_j\right)$, $\nu_T\left(v_i,w_j\right)$ and $\pi_T\left(v_i,w_j\right)$ are the grade of membership, non-membership and uncertainty of the ordered pair of points respectively in $\left[0,1\right]\in I$. Furthermore the condition $0\le\mu_T\left(v_i,w_j\right)+\nu_T\left(v_i,w_j\right)\le 1$ is follows and the degree of uncertainty is denoted by $\pi_T\left(v_i,w_j\right)=1-\left(\mu_T\left(v_i,w_j\right)+\nu_T\left(v_i,w_j\right)\right)$.

Definition 2. [13] Let T^{*} be an IFPR in $V\times W\times Z$ that is defined as

$$T^{*}=\left\{\left\langle\left(v_i,w_j,z_k\right),\mu_T\left(v_i,w_j,z_k\right),\nu_T\left(v_i,w_j,z_k\right)\right\rangle \mid \left(v_i,w_j,z_k\right)\in V\times W\times Z\right\} \tag{2}$$

where $\mu_T\left(v_i,w_j,z_k\right):V\times W\times Z\to I$ and $\nu_T\left(v_i,w_j,z_k\right):V\times W\times Z\to I$ follows the condition, $0\le\mu_T\left(v_i,w_j,z_k\right)+\nu_T\left(v_i,w_j,z_k\right)\le 1$ for every $\left(v_i,w_j,z_k\right)\in V\times W\times Z$.
If $\pi_T\left(v_i,w_j,z_k\right)=1-\left(\mu_T\left(v_i,w_j,z_k\right)+\nu_T\left(v_i,w_j,z_k\right)\right)$,
then $\pi_T\left(v_i,w_j,z_k\right)$ is the degree of uncertainty where the element $v_i\in V, w_j\in W$ and $z_k\in Z$ with $0\le\pi_T\left(v_i,w_j,z_k\right)\le 1$ and $\mu_T\left(v_i,w_j,z_k\right)+\nu_T\left(v_i,w_j,z_k\right)+\pi_T\left(v_i,w_j,z_k\right)=1$.

Space curves are one-dimensional objects in three dimensions which can be described mathematically either in Cartesian form or by two equations [14]. For example, for $(v,w,z)\in\mathbb{R}^3$, $w=v^2$ or $vz=w^2$. Next section will discussed and introduced intuitionistic fuzzy B-spline space curve.

INTUITIONISTIC FUZZY B-SPLINE SPACE CURVE

Space curve is a curve which may pass through any region of three dimensional space. Therefore, from Def. 6, Def. 7, and [15], IFB-SSC can be defined as follows:

Definition 3. Let $T_i^* = \{T_1^*, T_2^*, \ldots, T_{n+1}^*\}$ with $i = 1, 2, \ldots,\ n+1$ be intuitionistic fuzzy control point relation for 2D and 3D universe, then by blended it with B-spline function $N_i^k(t)$, intuitionistic fuzzy B-spline space curve is written as

$$S^*(t) = \sum_{i=1}^{n+1} T_i^* N_i^k(t) \tag{3}$$

with $t_{\min} \le t \le t_{\max}$ and $2 \le k \le n+1$ where C_i^* are the position vectors of $n+1$ control polygon vertices, and N_i^k are the normalized B-spline basis functions. The $N_i^k(t)$ is defined as

$$N_i^1(t) = \begin{cases} 1 & if \quad t_i \le t < t_{i+1} \\ 0 & otherwise \end{cases} \tag{4}$$

and

$$N_i^k(t) = \frac{(t - t_i)}{t_{i+k-1} - t_i} N_i^{k-1}(t) + \frac{(t_{i+k} - t)}{t_{i+k} - t_{i+1}} N_{i+1}^{k-1}(t) \tag{5}$$

IFB-SSC in (3) is parametric function consists of membership curve, non-membership curve and uncertainty curve and denoted as follows

$$S^\mu(t) = \sum_{i=1}^{n+1} T_i^\mu N_i^k(t) \tag{6}$$

$$S^\nu(t) = \sum_{i=1}^{n+1} T_i^\nu N_i^k(t) \tag{7}$$

$$S^\pi(t) = \sum_{i=1}^{n+1} T_i^\pi N_i^k(t) \tag{8}$$

From Def. 4 and intuitionistic fuzzy B-spline curve interpolation model introduced in [15], the spline space curve in helix form with membership and non-membership function for ($V \times W \times I$) is shown in Figure 1-3 and for ($(V \times W \times Z) \times I$) is shown in Figure 4.

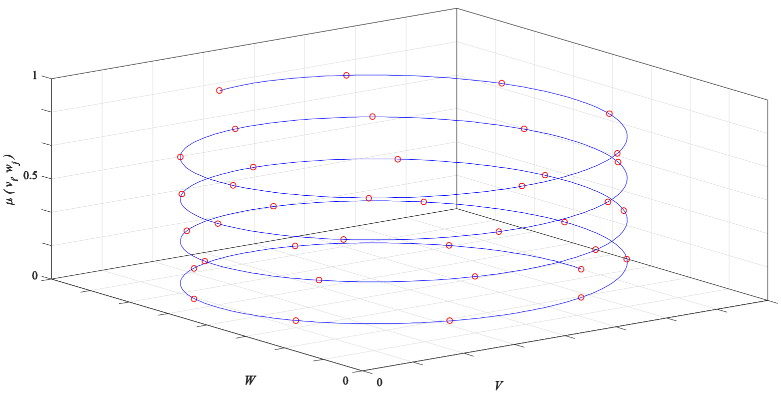


FIG. 1 Membership intuitionistic fuzzy B-spline space curve for $V \times W \times I$

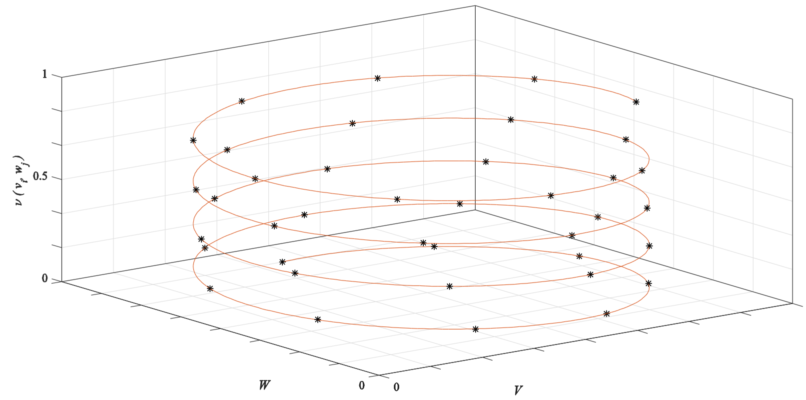


FIG. 2. Non-Membership intuitionistic fuzzy B-spline space curve for $V \times W \times I$

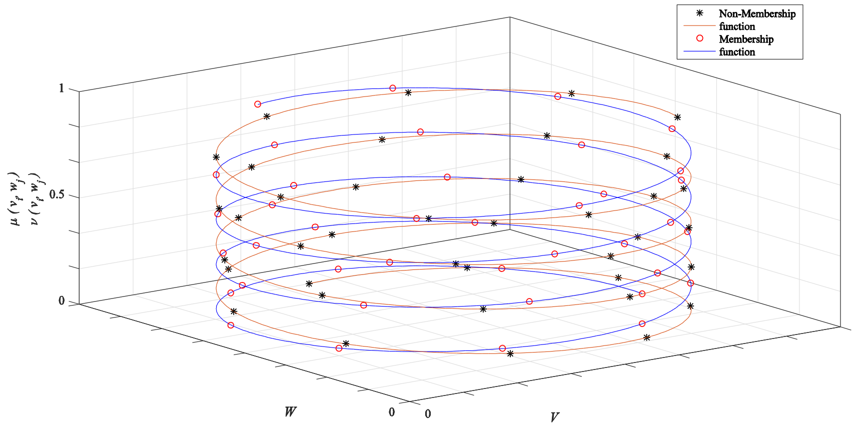
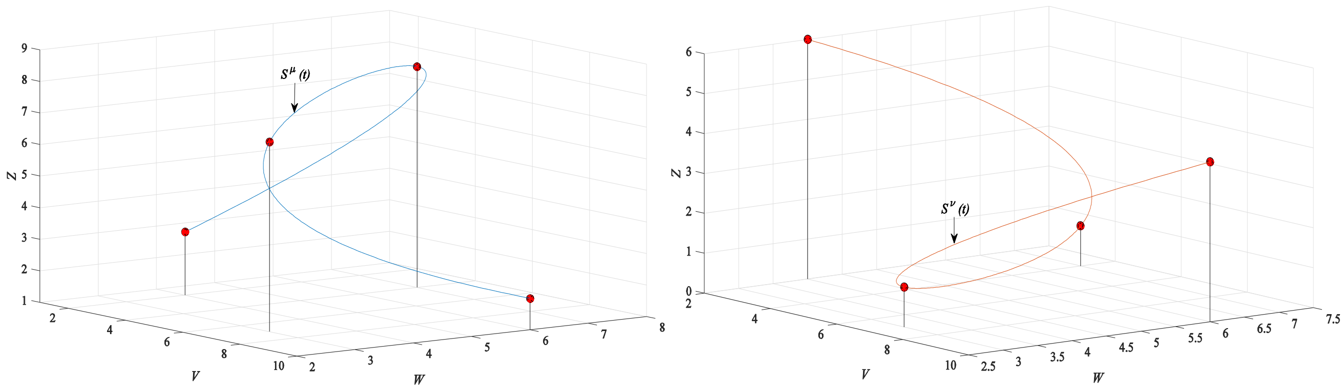
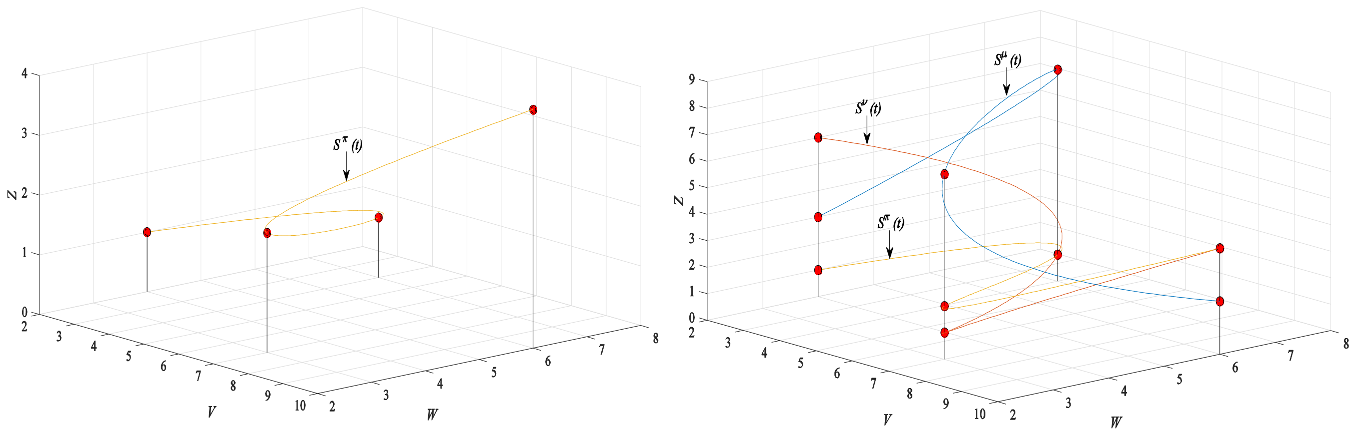


FIG. 3.intuitionistic fuzzy B-spline space curve for $V \times W \times I$



(a)

(b)



(c) (d)

FIG. 4. Intuitionistic fuzzy B-spline space curve for 3D ($(V \times W \times Z) \times I$) (a) Membership; (b) Non-Membership; (c) Uncertainty; (d) Intuitionistic fuzzy B-spline

Through Def. 4, some properties of IFB-SSC is obtained and listed out as below:

i. The IFB-SSC lies within the convex hull of its respective intuitionistic control polygon
ii. The IFB-SSC generally follows the shape of the intuitionistic control polygon
iii. The maximum order of the IFB-SSC equals the number of intuitionistic control polygon vertices where the maximum degree is less than one.
iv. The IFB-SSC exhibits the variation-diminishing properties.
v. Each basis function of IFB-SSC is positice or zero for all parameter values that is, $N_i^k(t)$.
vi. The sum of B-spline basis function of IFB-SSC for any value parameter *t* is $\sum_{i=1}^{n+1} N_i^k(t) \equiv 1$.

CONCLUSION

In this paper, IFB-SSC is introduced with some of its properties. IFB-SSC is a very useful approach and can be used in many fields such as real engineering designs, civil engineering designs, ship building, manufacturing, architectural designs, aeronautics, manufacturing and much more. Through the intuitionistic approach, more problems can be tackled with the existence of membership, non-membership and uncertainty functions. This IFS approach combine with B-spline tools can generate continuously differentiable smooth curve that can illustrate any studied problem completely. This method can be extend through B-spline surface, and Non-Rational Uniforms B-spline to obtain better results.

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