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Exploring the Roles of Visual Tools in Primary Mathematics to Achieve Learning Outcomes

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Abstract

The aim of this study was to investigate the roles of visual tools used by mathematics pre-service teachers in teaching primary mathematics, with a focus on achieving pupils' learning outcomes. Adopting a qualitative approach through a case study design, six mathematics pre-service teachers undergoing practicum attachments at various primary schools in Johor, Malaysia, were purposefully sampled as participants. Data collection included analysis of teaching plans, classroom observations, and interviews with the participants. The mathematical visual tools used for teaching primary mathematics were found to serve multiple roles: as inherent part of mathematics, concretisation of concepts, mitigate pupil difficulties, facilitate hands-on learning experiences, promote retention of information through repetitive use, and make mathematics interesting. In summary, the study underscores the critical role of visual tools in primary mathematics education and highlights the necessity for pre-service teachers to effectively integrate these tools to enhance pupils' learning outcomes.

Keywords: visualisation, visual tool, visual tool role, primary mathematics

VISUALISATION AND VISUAL TOOLS IN MATHEMATICS

There are two primary ways in which learners approach mathematical problems: verbal logical thinking and visual pictorial thinking [1]. Verbal logical thinking involves processing information through language, reading, discussion, and logical reasoning. Learners who excel in this mode of thinking are adept at analysing problems, identifying patterns, and applying logical principles to arrive at solutions. They rely on verbal explanations and written instructions to understand mathematical concepts and solve problems.

On the other hand, Thornton described that visual pictorial thinking involves processing information through visual representations, such as diagrams, graphs, or pictures [1]. Learners who favour this mode of thinking benefit from visualising concepts spatially and using visual tools to organise information and solve mathematical problems. Visual learners often have strong spatial reasoning skills and can quickly grasp geometric relationships or patterns within data. They may struggle with purely abstract or symbolic representations but thrive when concepts are presented visually. Hence, visual tools play an important role in making mathematical concepts more tangible, especially for learners who struggle with abstract thinking.

Visualisation is an essential aspect of learning mathematics that enables learners to make sense of mathematical concepts, relationships, and structures. It has long been incorporated in mathematics classrooms [2], even dating back since the Piagetian era [3]. In fact, the integration of visualisation in learning mathematics still remains relevant particularly among primary mathematics education. Its relevance in the mathematics classrooms is evident in the form of digital, as well as, non-digital visual tools.

Many studies used Arcavi's definition to define visualisation as an intervening conceptual structure. It refers to the ability, process, and outcome of creating, interpreting, using, and reflecting upon pictures, images, and diagrams either in the mind, on paper, or with technological tools, in order to communicate information, think through, and develop ideas [4]. Visual tools can be presented in two forms: the physical resemblance, where the visual tool directly resembles the actual object or concept, and the structural resemblance, where the characteristics of the concept are abstractly represented [5]. For instance, a modelling problem might depict the height of a building using the length of a vertical line as a physical resemblance. Alternatively, the height of a group of objects could be displayed using a graph using the structural resemblance.

Visualisation is now acknowledged as a crucial part of cognitive processes and is no longer limited to being merely illustrative. This idea of visualisation is consistent with Bruner's long-standing emphasis on the importance of representation to facilitate conceptual understanding [6]. According to his theory, it is essential to use the right form of representation to ensure that a learner's conception is accurate. Different concepts may require different modes of representation for effective understanding, namely enactive, iconic, or symbolic. By leveraging the right tools, teachers can promote deeper conceptual understanding, enhance problem-solving skills, and foster mathematical proficiency. The importance of visual tools in learning mathematics, particularly primary mathematics lies in its ability to make abstract concepts tangible [2, 7, 8, 9, 10], foster conceptual understanding [1, 2, 8, 9], cater to diverse learning styles [1, 2, 10], and develop engagement and interest in mathematics [2, 7, 9].

Despite the recognition of the importance of visual tools in teaching mathematics, there exists a gap in teachers' understanding, proficiency, and utilisation of a wide range of visual tools [11]. While research indicates that appropriate utilisation of representations in mathematics teaching is crucial for student learning [10, 12], and that visual tools and digital visualisation technologies are promoted for their potential to assist student understanding in mathematical concepts [2, 9, 11, 12], there is limited understanding of how teachers, particularly, pre-service teachers integrate visual tools for teaching mathematics. Besides that, there remains a need to understand how visual tools selected by pre-service teachers play their roles during mathematics lesson activities for effective learning, which is translated into learning outcomes. Therefore, this study aimed to understand in-depth the roles of visual tools used by mathematics pre-service teachers when teaching primary mathematics to achieve pupils' learning outcomes.

METHODOLOGY OF THE STUDY

This study adopted the qualitative approach through a case study design. Purposeful sampling method was used where the participants of the study consisted of six mathematics pre-service teachers who underwent practicum attachments at various primary schools around the state of Johor across different grade levels. The number of research participants was determined on the basis that the data collection had reached a point where additional input from new participants no longer generated new information, themes, or understanding of how visual tools played their role when incorporated into mathematics learning activities. By reaching data saturation with the six participants, it was ensured that this case study had sufficient depth and richness of data to adequately address the research objective.

Data was collected through three sources, documents, classroom observations, and interviews which spanned from August 2023 to April 2024. Firstly, the participants' teaching plans were analysed. The main purpose of this step was to establish if the participants incorporated visual tools in their lesson. The visual tools in this study were focused on visual tools that are directly mathematical related. The topic, objective(s), content standard, and the learning standard(s) of the lesson, as well as the grade level were also recorded for the purpose of identifying emerging themes in the participants' decision-making processes when selecting visual tools for teaching.

Next, a series of classroom observations were conducted using the framework in a study by Dufour-Janvier, Bednarz, and Belanger as the basis to analyse and understand the roles played by particular visual tools when integrated into the mathematics learning activities to achieve particular mathematics learning outcomes. According to this framework, the tools that teachers used in teaching could be (i) an inherent part of mathematics, where concepts involved cannot be studied without the aid of the particular representations, (ii) a concretisation of a concept, (iii) to mitigate difficulties faced by pupils, and (iv) make mathematics more interesting [13]. While these four roles formed the basis of the analysis, other emerging themes that emerged during analyses were also examined.

After the observation was conducted, the participants were interviewed using questions guided from the protocol by Schmitz and Eichler related to how and why they use the particular visual tools in their classrooms [14]; their aims for the lesson, their perspective on the pupils' learning process using the visual tools, and their use of technology (if applicable). Thematic analysis was used to identify patterns, themes, and insights from the interview transcripts to understand the decision-making processes employed by the participants when they selected visual tools for teaching specific mathematics topics.

THE ROLES OF VISUAL TOOLS IN PRIMARY MATHEMATICS LEARNING

The observation and analyses found that out of 16 classroom observations conducted throughout the data collection period, 12 mathematics lessons incorporated mathematical visual tools. The mathematical visual tools integrated in the lessons observed dominantly played two roles, namely, as an inherent part of mathematics, and to make mathematics more interesting.

It was evident among five of the participants, Teacher A, P, S, T, and W that they integrated various mathematical visual tools as an inherent part of mathematics. For example, Teacher W used tables and bar charts as visual tools to teach Data Handling to Year Four pupils aligning with Content Standard 8.1, Learning Standards 8.1.1 and 8.1.2, which required pupils to construct bar charts for ungrouped data and interpret them. Another instance was Teacher S, who used the hundred square grids and number lines to teach Decimals to Year Three pupils on Content Standard 3.1, Learning Standard 3.2.3 that focuses on comparing the values of two decimal numbers up to two decimal places using hundred square grids and number lines. In simpler terms, these participants used mathematical visual tools to teach these content standards because the Malaysian Mathematics Standards-based Curriculum and Assessment Document (DSKP) outlines them as a fundamental aspect for teaching the content.

It is also important to highlight that these mathematical visual tools used by Teachers W and S were in the form of digital tools with various colours and animations to engage their pupils and make the lesson more interesting. In fact, all participants used their visual tools in a creative way that was obviously purposed to make their lessons more interesting. Teacher P used an online number generator to teach Content Standard 1.3, Learning Standard 1.3.1 to Year Six pupils that focuses on classifying prime and composite numbers within 100. The online number generator appeared to excite the pupils in anticipating which number would appear on the smartboard in the classroom for them to determine if it is a prime or composite number. Meanwhile, Teacher T taught the same learning standard to her group of pupils using the treasure hunt theme throughout the lesson. She prepared number cards hidden in 'treasure chest' envelopes around the classroom so that the pupils would need to find them and determine if the number is a prime or composite number.

Besides that, there were also mathematical visual tools used in lessons to enable pupils to concretise particular mathematical concepts. For example, the digitally constructed bar charts were projected on the smartboard by Teacher W to teach Year Four pupils to interpret them (Content Standard 8.1, Learning Standard 8.1.2). The pupils compared the height of the bars on the chart to answer questions posed by the teacher related to student enrolment in school uniform clubs. Since Mandarin is the language of instruction at this school, the questions were asked in that language. The English translation of an excerpt from the lesson conversation is displayed as follow.

W	:	5) 最多学生和最少学生参加的制服团体的差是? (Translation: What is the difference between the uniform club with the most and least students?)
Pupil 3	:	14, Teacher.
W	:	How do you find the answer?

Pupil 3 : 46 minus 32, Teacher.

W : How do we know which number to subtract?

Pupil 1 : Uniform club with most students and least students.

W : How do we know which uniform club has the most students?

Pupils : The tallest bar

W : Correct. How do we know which uniform club has the least number of students?

Pupils : Shortest bar

Another scenario where mathematical visual tools were used to concretise mathematical concepts was evident when teaching converting time units between days and weeks by Teacher A to Year Four pupils (Content Standard 4.4, Learning Standard 4.4.2). Teacher A displayed the calendar application on his laptop to the class, and pupils were given a worksheet containing conversion problems as depicted in Figure 1. Teacher A used the calendar application to tally the days based on the week as he went through the problems listed in the worksheet, making connections between seven days and a week. Then, he related it to multiples of seven, subsequently performing calculations using standard form.

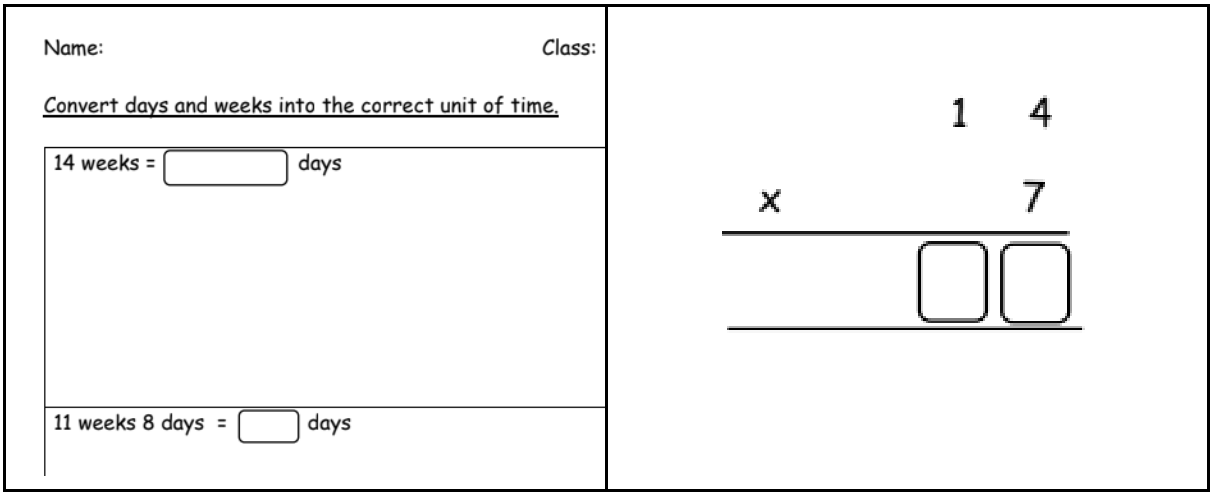

Figure 1

Besides using technological visual tools, concrete manipulatives were also observed to be visual tools used to concretise mathematical concepts. For instance, Teacher S brought donuts to teach her Year Three pupils how to identify proper fractions as part of a whole (Content Standard 3.1, Learning Standard 3.1.1). She served a donut to each group of pupils on a plate and a plastic knife. With varying numbers of members, each group was instructed to share the donut equally. Next, Teacher S asked a few oral questions to each group such as, "How many donuts do you have?" and "How many pieces did each of you get?". She then proceeded to ask the pupils to state, in fractions, the portion of donuts they had received, making connections between the concept of part of a whole, the terms numerator and denominator, and this activity.

Teacher K also used concrete manipulatives to concretise the basic facts of division for her Year Two pupils (Content Standard 2.4, Learning Standard 2.4.1). She printed a few rabbits and carrots on cards and requested a few pupils to feed the rabbits equally. She then asked them questions like, "How many rabbits are there?", "How many carrots are there altogether?", and "If you feed the rabbits with an equal number of carrots, how many carrots did each rabbit get?". She continued by making connections between the rabbit feeding activity and the concept of fair sharing.

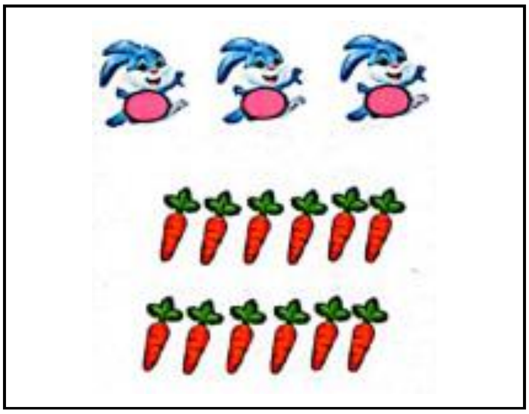

Figure 2

It was also evident that mathematical visual tools were used to mitigate difficulties when learning particular concepts in mathematics. This was particularly evident when Teacher S used real donuts to teach her Year Three pupils how to identify proper fractions as part of a whole (Content Standard 3.1, Learning Standard 3.1.1), as well as Teacher W who used a 1-100 number chart to assist her Year Five pupils to identify prime numbers within 100 (Content Standard 1.2, Learning Outcome 1.2.1). The activity of dividing the donuts into equal portions to distribute among the group members enabled the pupils to count each cut piece individually and determine the numerator and denominator. Additionally, the pupils were also able to comprehend that there are multiple ways to depict the concept of part of a whole with each group having a donut while having to share it with different number of group members.

Meanwhile, Teacher W engaged her pupils in a game called "Bomb the Prime Number," in which they had to take turns counting out loud all the numbers from one to a hundred. The pupils would shout "Bomb!" rather than the number whenever they came to a prime number. In the event that they are unable to "bomb" the prime number, the subsequent student will carry on. The students first seemed to be having trouble with the game, as they either missed a few numbers or failed to shout "Bomb!" Teacher W projected the 1-100 number chart to help with this challenge. The pupils were able to play the game more effectively after that, clearly achieving the learning objective through the game.

Apart from the four roles described in the framework by Dufour-Janvier, Bednarz, and Belanger, this study observed two other roles of mathematical visual tools. Firstly, the integration of mathematical visual tools enabled hands-on experience. This proved evident in Teacher A's class, in which he had his Year Four pupils count using the calendar to demonstrate how to convert time units between days and weeks. The donut activity by Teacher S also helped her pupils to concretise the concept of fraction through hands-on experience.

The lesson observations also found that mathematical visual tools were also used for retention of information when used repetitively. This role of integrating visual tools was particularly evident in Teacher P's Year Six class on classifying prime and composite numbers within 100 (Content Standard 1.3, Learning Standard 1.3.1). He projected a checklist table as depicted in Figure 3 to serve as a guide for his pupils to identify if the number posed is a prime or composite number.

Figure 3

The checklist table involved three criteria, namely, (i) the number is greater than 1; (ii) the number is divisible by one and itself; (iii) the number is divisible by other integers. As the pupils progressed to identify each number posed by Teacher P, they demonstrated their reference to the checklist, thus, achieving the learning objective.

DISCUSSION & CONCLUSION

This paper analysed mathematics pre-service teachers' integration of mathematical visual tools and their decision-making processes in selecting them for teaching primary mathematics to achieve pupils' learning outcomes. All the participants used mathematical visual tools to teach various learning areas in primary mathematics across grade levels. Despite their usage of these tools appear to be similar on the surface, in-depth analyses indicated various roles of mathematical visual tools as the participants integrated them in their lessons.

Although the elements described in the framework by Dufour-Janvier, Bednarz, and Belanger are generic in nature and not specifically designed for a mathematics classroom, each element was clearly evident in the mathematics classrooms that were involved in this study. Therefore, we can perceive that visual tools in a primary mathematics lesson do play the roles as described in the study conducted by Dufour-Janvier, Bednarz, and Belanger. In fact, previous research has also stressed the importance of visual tools' ability to make abstract concepts tangible [2, 7, 8, 9, 10]. By engaging with visual tools, pupils can visualise relationships, patterns, and problem-solving strategies, thereby bridging the gap between abstract concepts and real-world applications.

The mathematical visual tools used by the participants in this study also appeared to facilitate concretisation of concepts. This finding has been acknowledged for a long time as a function of visual tools [1, 2, 8, 9]. The results of the analyses also showed that, in line with previous studies [2, 7, 9], the mathematical visual tools were able to develop engagement and interest in primary mathematics.

Additionally, this study also revealed some elements that could add on towards the roles of visual tools specifically in the learning of mathematics to assist pupils to achieve the learning outcomes. The integration of mathematical visual tools enabled hands-on experience among pupils, allowing them to interact directly with mathematical concepts in a tangible way. For example, the donut sharing activity allowed the pupils to manipulate it to understand the abstract concept of fraction as part of a whole. Hence, when pupils have hands-on experience with visual tools, they can visualise and manipulate mathematical ideas, leading to deeper comprehension. This association supports the notion that visualisation can function as an efficient tool for generating images and assisting learners in comprehending abstract concepts through a concrete or pictorial approach to assist exploration and testing of hypotheses, which is another aspect of constructivism that is being advocated in current pedagogies [15].

Allowing hands-on experience with visual tools enhances pupils' problem-solving skills, as it enables them to explore and experiment with mathematical concepts in a dynamic way [16]. When faced with mathematical problems, pupils can use visual representations to model situations, test hypotheses, and develop solutions. Hands-on activities using visual tools can increase their engagement and motivation when learning mathematics. When pupils are actively involved in their learning through hands-on experiences, they are more likely to be attentive, curious, and motivated to participate [16].

Besides that, repetitive use of visual tools aided in the retention of mathematical information. While traditional teaching approaches are characterised by drilling and repetition [8], it is essential to draw attention to the role of mathematical visual tools as being able to aid pupils in their conceptual understanding of the topic being learned. For example, when Teacher P repeatedly used the checklist table as a guide to assist his pupils to classify the type of number posed, it helped to reinforce their understanding and memory of key concepts. Additionally, the repetitive use of visual tools reinforces conceptual understanding over time, allowing pupils to make connections between different mathematical concepts and apply them in various contexts. This role ensures all pupils have the opportunity to engage with the mathematical concept at their own pace and level. An advanced pupil may be able to immediately classify the number without or with minimal guidance of the checklist table, while a weaker pupil may refer to it to classify the number posed by the teacher. Hence, it helps to promote equity in the classroom particularly when classrooms are not streamed according to pupils' level of achievement.

In summary, this study highlights the importance of integrating mathematical visual tools in primary mathematics education. It is important to emphasise that these visual tools should go beyond mere enthusiasm or for the sake of education policy on digital utilisation in the classroom. Instead, it should also focus their significance in the following aspects: (i) as an inherent part of mathematics; (ii) as concretisation of a concept; (iii) to mitigate pupil difficulties (iv) in facilitating hands-on learning experiences; and (v) in promoting the retention of information through repetitive use.

REFERENCES

[1] Thornton, S. (2001). A Picture is Worth a Thousand Words. *Proceedings of the International Conference of the Mathematics Education into the 21st Century Project*, 251–256.

[2] Svitek, S., Annuš, N., & Filip, F. (2022). Math Can Be Visual - Teaching and Understanding Arithmetical Functions through Visualization. *Mathematics*, *10*, 2656. https://doi.org/10.3390/math10152656

[3] Gunčaga, J., & Žilková, K. (2019). Visualisation as a method for the development of the term rectangle for pupils in primary school. *European Journal of Contemporary Education*, *8*(1), 52–68. https://doi.org/10.13187/ejced.2019.1.52

[4] Arcavi, A. (1999). The Role of Visual Representations in the Learning of Mathematics. *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 55–80.

[5] Schoenherr, J., & Schukajlow, S. (2024). Characterizing External Visualization in Mathematics Education Research: a Scoping Review. *ZDM - Mathematics Education*, *56*(1), 73–85. https://doi.org/10.1007/s11858-023-01494-3

[6] Bruner, J. S. (1966). *Toward a Theory of Instruction*. Harvard University Press.

[7] Bednarz, N., & Janvier, B. (1987). Reconstruction of a Transformation: A Problem in Mathematics. *Biennial Meeting of the Society for Research in Child Development*. Baltimore.

[8] Chatima, S. (2021). *Visualisation Processes in Teaching Patterns and Their Generalisation: Perceptions and Experiences from Senior Phase Mathematics Teachers*. [Master thesis, Rhodes University].

[9] Embong, Z., Mohamed Adnan, N. S., Saidin, N., & Ibrahim, H. (2019). Effects of Visual Approach in Teaching Mathematics for Malaysian Secondary School: A Case Study. *Journal of Human Development and Communication*, *8*, 97–106.

[10] Lasiun, M. (2016). Keberkesanan Kaedah Visualisasi: Meningkatkan Keupayaan Menyelesaikan Masalah Matematik Berayat. *International Seminar on Generating Knowledge Through Research, UUM-UMSIDA*, *1*(1). https://doi.org/10.21070/picecrs.v1i1.542

[11] Smith, C., & Cekiso, M. (2020). Teachers' understanding and use of visual tools in their numeracy classrooms: A case study of two primary schools in gauteng. *South African Journal of Childhood Education*, *10*(1), 1–8. https://doi.org/10.4102/SAJCE.V10I1.887

[12] Mainali, B. (2021). Representation in teaching and learning mathematics. *International Journal of Education in Mathematics, Science and Technology*, *9*(1), 1–21. https://doi.org/10.46328/ijemst.1111

[13] Dufour-Janvier, B., Bednarz, N., & Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. *Problems of representation in the teaching and learning of mathematics*, 109-122.
[14] Schmitz, A., & Eichler, A. (2015). Teachers' Individual Beliefs About the Roles Of Visualization In Classroom. *9th Congress of the European Society for Research in Mathematics Education (CERME)*, 1266–1272. https://hal.science/hal-01287355
[15] Johnny, J. (2023). Visualising Mathematical Functions: Learning Limit and Continuity Via Plot-It. *International Journal of Research in Teacher Education (MIJORiTE)*, *4*, 11–26.
[16] Schoenfeld, A. H. (2011). *How We Think: A Theory of Goal-Oriented Decision Making and its Educational Applications*. Routledge.