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Examining the Thermal Dynamics of Coffee Cups: Evaluating the Impact of Cup Material on Thermal Characteristics via Numerical Simulations

Sabastine Emmanuel[1,2], Saratha sathasivam[1*], Siti Fatimah Azzahrah Binti Azhar[1], and Nur Elya Nadhirah Binti Che Mazhaimi[1]

[1]School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Pulau Pinang, Pulau Pinang, Malaysia
[2]Department of Mathematics, Federal University Lokoja, Nigeria

**Corresponding author:* saratha@usm.my

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Abstract

This work examines the thermal diffusivity properties of four different materials, ceramic, stainless steel, plastic, and glass, using the one-dimensional heat equation's Initial Boundary Value Problem (IBVP) framework. We investigate the transient thermal behaviour of these materials using numerical techniques like the Crank-Nicolson method and the explicit FTCS (Forward-Time Central-Space) method. We perform simulations and analyse heat transfer dynamics and temperature distributions using Python implementations with uniform step sizes. According to our research, there are notable differences in the thermal diffusivity performance of the materials, with stainless steel showing better conductive qualities. Furthermore, a look at the midpoint temperature profiles of the cups provides information on the thermal and temporal dynamics. A comparison of the Crank-Nicolson and FTCS approaches shows how effective the latter is in producing precise and stable solutions. The study contributes to a deeper understanding of material thermal properties and numerical methods' suitability for simulating heat transfer phenomena.

Keywords Heat transfer, Thermal performance, Heat equation, Crank-Nicolson method, Thermal conductivity.

INTRODUCTION

The study of thermal diffusivity in materials is crucial for understanding heat transfer processes in various applications, ranging from industrial manufacturing to environmental engineering.Heat transfer is a continuous medium that transmits heat in solid, liquid and gas. This heat transfer can be stimulated using a heat equation as a partial differential equation. The heat equation describes the distribution of temperature or heat fluctuation in a particular transmission area over time, also explained as the basic mathematical theory of thermal conductivity [1] The heat equation is a second-order parabolic partial differential equation, which can be addressed using various numerical techniques [2]. It's essential to showcase the solution of this one-dimensional heat equation as it represents the transfer of heat from a solid to a liquid. In this study, we stimulate the heat transfer in cups of coffee with different types of cup material and solve them using numerical methods of Finite Difference Time-Stepping (FTCS) and Crank-Nicolson methods. The purpose of solving the heat equation is to investigate the thermal conductivity of distinct materials of cups of ceramic, stainless steel, plastic and glass that transfer the heat from hot coffee to the cups throughout the height of the cups.

Comprehending the phenomena of heat transfer is crucial for enhancing the thermal efficiency of diverse systems, encompassing commonplace items such as coffee cups. Thermal analysis is based on the understanding of heat conduction in solids, as provided by seminal publications like those by [3]. The fundamentals of heat conduction, which are essential to simulating heat transmission in coffee cups, are thoroughly covered in these texts [4] provide a thorough introduction to the heat transfer principles and go into additional detail. Their research is a useful tool for learning about the processes controlling heat transfer and the variables affecting thermal behaviour in various materials. In order to analyze complicated systems, numerical approaches are essential for simulating the transfer of heat processes. [5] presents methods for discretizing and computationally solving the governing equations, shedding light on computational energy transfer and fluid flow. This study is an invaluable resource for applying numerical methods to heat transfer models. [6] offers an in-depth exploration of the fundamental principles underlying heat transfer phenomena, including conduction, convection, and radiation. Through insightful examples and practical applications, the text simplifies complex concepts, enabling readers to grasp the intricacies of heat transfer and its implications for engineering systems with greater clarity.

Numerical methods, particularly finite difference methods (FDM) and finite element methods (FEM) have become increasingly popular for analyzing thermal diffusivity in materials. The explicit FTCS method and the implicit Crank-Nicolson method are two commonly employed techniques for solving the one-dimensional heat equation. The use of finite difference approaches in heat transfer analysis is covered by [7], who also offers computational methods and algorithms for resolving transient and steady-state heat conduction issues. [8] discusses computational heat transfer, which investigates sophisticated numerical methods for simulating heat transport events. The book provides a thorough overview of computational methods for studying heat transport problems, including finite element, finite volumeandfinitedifferenceapproaches

Recent studies conducted by [9] provide an in-depth exploration of heat and mass transfer principles, offering valuable insights into thermal performance determinants across diverse systems and elucidating the underlying mechanisms of heat transfer. Conduction, convection, and radiation are all covered in [10] thorough textbook on heat transfer. For both researchers and students, the text is an invaluable resource because it provides thorough explanations of heat transfer mechanisms together with real-world examples.

For an optimal coffee-drinking experience, maintaining the ideal temperature of the beverage is paramount. The rate of coffee cooling is influenced by multiple factors, including the thermal properties of the cup material. An in-depth understanding of heat transfer dynamics within different cup materials can drive the development of more effective coffee cups capable of sustaining temperature levels for longer durations. This research aims to investigate how cup material affects the thermal performance of coffee by simulating heat transfer within coffee cups. Analyzing the diffusion rate of coffee temperature across various materials commonly used for coffee cups, such as ceramic, glass, stainless steel, and plastic, is a focal point.

The primary goal is to scrutinize and compare the heat transfer characteristics of coffee cups made from different materials. By employing the Finite Difference Method (FDM), specifically leveraging the Forward Time Central Space (FTCS) and Crank-Nicolson methods, we intend to simulate the heat conduction process within the coffee cups. Through these simulations, we seek to dissect and compare the thermal efficacy of the cups, providing invaluable insights into how different cup materials impact the rate of temperature variation in the coffee.

MATERIALS AND METHODS

We focus on understanding the effects experienced by coffee drinkers. Understanding the thermal performance of coffee cups can lead to the development of cups that keep coffee at an optimal temperature for a longer time. This is possible by identifying cup materials that are better at retaining heat, and able to reduce the need for reheating to maintain the coffee temperature. This research can assist coffee cup manufacturers, designers, and consumers in deciding cup material selection based on heat retention properties.

Figure 1: Heat Retention in Various Cup Materials

This research uses Finite Difference Time Stepping (FTCS) and Crank-Nicolson to model the heat diffusivity process in coffee cups. By comparing the thermal diffusivity performance of different cup materials, this research can provide insights into the advantages and limitations of each material. The result (performance) will be compared. This information can guide the development of improved coffee cup designs and inform consumers about the best material choices for their preferences.

Suppose that the heat within the coffee is confined to the boundaries of the cup, with the cup's temperature remaining constant regardless of its position. Let the height of the cup be denoted by L. The position along the vertical axis of the cup, ranging from the bottom to the top, is represented by x in meters. The temperature within the cup is a function of both position xx and time tt, forming a relationship denoted by $u(x,t)$. We will employ the Forward-Time Central-Space (FTCS) and Crank-Nicolson methods to analyse heat diffusion using Python. Common cup materials such as ceramic, stainless steel, plastic, and glass will be considered. By investigating the effect of heat on different thermal diffusivities, we aim to assess the accuracy of both methods. It is noted that the FTCS method is conditionally stable; thus, this research also seeks to validate this assertion.

We will also perform a sensitivity analysis to understand further how changes in initial conditions and thermal qualities affect the outcomes. When applied to real-world situations where material qualities might not be fully known, this study will aid in determining how resilient each method is. We can evaluate how these modifications affect the accuracy and stability of the FTCS and Crank-Nicolson techniques by methodically adjusting factors Like thermal diffusivity and initial temperature distribution. Lastly, a thorough discussion of these approaches' Python implementation will be given, emphasising the difficulties and solutions related to computing. We will investigate the effectiveness of various optimisation strategies and numerical solvers for managing large-scale simulations. To ensure consistency and promote future developments in thermal analysis, we hope to give researchers the tools they need to apply these techniques to their heat diffusion problems by offering a thorough overview of the coding procedure.

Heat Equation

The heat equation is a fundamental partial differential equation that describes how heat diffuses through a material over time. It is widely used in physics and engineering to model temperature distribution and heat transfer processes. The general form of the one-dimensional heat equation is:

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u \qquad (1)$$

Where:

- $u(x,t)$ is the temperature at the position x and time t,
- α is the thermal diffusivity of the material, and
- $\frac{\partial u}{\partial t}$ represents the rate of change of temperature with respect to time,

- $\nabla^2 u$ represents the spatial second derivative of temperature, indicating how temperature changes with position.

In the FTCS scheme, the time derivative is approximated using a forward difference and the spatial derivative using a central difference:

$$\frac{u_i^{n+1}-u_i^n}{\Delta t} = \alpha \frac{u_{i-1}^n-2u_i^n+u_{i+1}^n}{(\Delta x)^2} \quad (2)$$

On the other hand, the implicit Crank-Nicolson technique combines the forward time step with the average of the central difference between the current and subsequent time steps

$$\frac{u_i^{n+1}-u_i^n}{\Delta t} = \frac{\alpha}{2}\left(\frac{u_{i-1}^{n+1}-2u_i^{n+1}+u_{i+1}^{n+1}}{(\Delta x)^2} + \frac{u_{i-1}^n-2u_i^n+u_{i+1}^n}{(\Delta x)^2}\right) \quad (3)$$

This results in a system of linear equations that can be written in matrix form and solved

The analytical solution for the one-dimensional heat equation, considering both initial and boundary conditions, is derived by setting up an initial boundary value problem (IBVP). Through this formulation:

$$u_t(x,t) = \alpha u_t \text{ for } 0 \le x \le L \text{ and } t > 0 \quad (4)$$

With initial and boundary conditions

$$\left.\begin{matrix} u(x,0) = f(x)\ for\ 0 \le x \le L \\ u(0,t) = u(L,t) = 0\ for\ t > 0 \end{matrix}\right\} \quad (5)$$

Then, we can obtain the exact solution by applying the separation variable as follows, consider the heat equation

$$\begin{aligned} &u(x,t) = X(x)T(t) \\ &\Rightarrow X(x)T'(t) = kX''(x)T(t) \end{aligned} \quad (6)$$

Then divide the equation by $kT(t)$ and $X(x)$, we obtained

$\frac{T\prime(t)}{kT(t)} = \frac{X\prime\prime(x)}{X(x)}$ and we let μ be a constant such that $\frac{T\prime(t)}{kT(t)} = \mu$ and $\frac{X\prime\prime(x)}{X(x)} = \mu$ this implies that

$\Rightarrow X'' - \mu X = 0$ and $\Rightarrow T' - \mu KT = 0$. Three cases arise, these are

Case 1: $\mu = 0$

We obtain the equation $X'' = 0$ which is a solution of general form $X(x) = Ax + B$ with the boundary condition $X(0) = B = 0$ and $X(L) = A(L) + B$. Hence, we obtain zero solution because $A = B = 0$.

Case 2: $\mu = \lambda^2$

We obtain $X'' - \lambda^2 X = 0$ and $T' - \lambda kT = 0$. Hence the general solution $X(x) = C_1 e^{\lambda x} + C_2 e^{-\lambda x}$
And $T = Ce^{\lambda^2 x}$ from boundary condition, we obtain $X(0) = C_1 + C_2 = 0$ and $X(L) = C_1 e^{\lambda x} + C_2 e^{-\lambda x}$. $\Rightarrow$ $C_1(e^{\lambda x} + e^{-\lambda x})$, which implies that $C_1 = 0$ and $C_2 = 0$. Hence $X(x) = 0$

Case 2: $\mu = -\lambda^2$

In this case, the obtained equation is given
$X'' - \lambda^2 X = 0$ and $T' - \lambda^2 kT = 0$.

Therefore for the general equation

$X(x) = Acos\lambda x + Bsin\lambda x$ and $T = Ce^{\lambda^2 kt}$, hence by using the boundary condition $X(L) = 0, X(0) = 0, \Longrightarrow A = 0$, $Bsin\lambda L = sin\lambda L = 0$

Implies $\lambda L = n\pi$n

Hence, $\lambda^2 = \frac{n^2\pi^2}{L^2}, \Longrightarrow X(x) = Bsin\left(\frac{n\pi x}{L}\right)$. Let $B = 1, \mu_n = \frac{n^2\pi^2}{L^2}$ and $X_n(x) = sin\left(\frac{n\pi x}{L}\right)$ then μ_n is an eigenvalue of the Sturn-Liouville problem with $X_n(x)$ as the corresponding eigenfunction. Notes that the set of eigenvalues and eigenfunction for $n \to 1$ to $n \to \infty$ encompasses the complete solution to the entire solution of the Sturn-Liouville problem. Hence, the solution is

$$\mu_n(x,t) = e^{\frac{n^2\pi^2}{L^2}} sin\left(\frac{n\pi x}{L}\right), u(x,t) = \sum_{n=1}^{\infty} B_n e^{\frac{n^2\pi^2}{L^2}} sin\left(\frac{n\pi x}{L}\right) \quad (7)$$

As the heat equation. Using the initial condition of the coefficient B_n

$$u(x,0) = f(x) = \sum_{n=1}^{\infty} B_n sin\left(\frac{n\pi x}{L}\right),\ B_n = \frac{2}{L}\int_0^L sin\left(\frac{n\pi x}{L}\right) dx \quad (8)$$

gives the analytic solution for the heat equation

Thermal Diffusivity (α)

The one-dimensional heat equation is described as: $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$

Thermal diffusivity, α, is a fundamental characteristic of materials that describes the rate at which heat can be transferred through a substance. It measures how efficiently a material can conduct heat to its capacity to store and retain thermal energy. Thermal diffusivity provides insights into the material's ability to rapidly distribute and equalise temperature changes.

These are the list of thermal diffusivities that will assist in the research.

Table 1. Thermal Diffusivity for various material

Material	k (m^2 / s)
Ceramic	**$1.5 \times 10^{-6}\ m^2/s$**
Stainless Steel	**$4.2 \times 10^{-6}\ m^2/s$**
Plastic	**$1.0 \times 10^{-7} m^2/s$**
Glass	**$7.0 \times 10^{-7} m^2/s$**

Numerical Simulation

Two common finite difference methods used for solving heat equations are the explicit method of center-finite difference technique and the Crank-Nicolson finite difference method. These methods are employed to derive numerical solutions for the heat equation.

Considering a cup with dimensions $[0, L] \times [0, T]$, divided into a finite number of nodes (x_i, t_n) we denote $u_t^n = u(x_t, t_n)$ as the numerical solution at $x \in [0, L]$ and $t \in [0, T]$.

Finite Difference Time-Stepping (FTCS) Methods:

FTCS are an explicit centred-difference scheme, that discretizes the equation using a forward difference method of first order in time combined with a centred difference approach of second order in space. Hence it is effective to solve the heat equation.

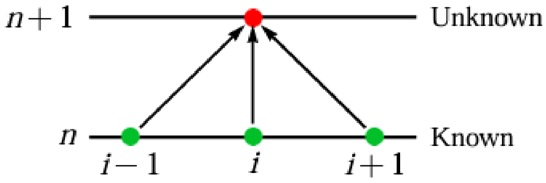

Figure 2. Computational molecule for the explicit FTCS scheme

To approximate the time derivative based on the figure. We use the forward difference equation. The forward difference in time:

$$\frac{\partial u}{\partial t}(x_i^n) \approx \frac{u_i^{n+1} - u_i^n}{\Delta t} \tag{9}$$

Which has an error of $O(\Delta t)$ and the centred difference in space:

$$\frac{\partial^2 u}{\partial t^2}(x_i^n) \approx \frac{1}{(\Delta x)^2}[u_{i-1}^n - 2u_i^n + u_{i+1}^n] \tag{10}$$

Substituting both equations (9) and (10) into the heat equation, the explicit centered-difference scheme for the one-dimensional heat equation is derived as follows:

$$u_i^{n+1} = (1 - 2r)u_i^n + r[u_{i-1}^n + u_{i+1}^n] \tag{11}$$

Where $r = \frac{k\Delta t}{(\Delta x)^2}$ (12)

Crank-Nicolson Method:

The modelling experiment requires time-accuracy, hence Crank-Nicolson scheme has significant advantages over FTCS. The method is a numerical technique well-known for its implicit characteristics and its second-order accuracy in both spatial and temporal domains. It provides an effective approach for solving equations by employing a combination of forward and backward differencing at the midpoint of the time increment. This technique ensures a highly accurate solution. It is illustrated in the figure below.

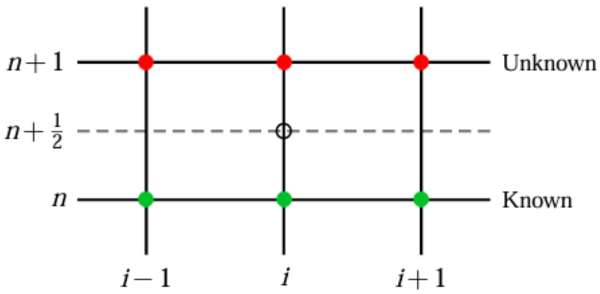

Figure 3. Grid point for the Crank-Nicolson scheme

By approximating the temporal first derivative at $t^{l+1/2}$ by

$$\frac{\partial T}{\partial x} \approx \frac{T_i^{l+1} - T_i^l}{\Delta t} \tag{13}$$

By averaging the difference approximation at the start and end of the time interval, we derive the second derivative with respect to space:

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{1}{2}\left[\frac{T_{i+1}^l - 2T_i^l + T_{i+1}^l}{(\Delta x)^2} + \frac{T_{i+1}^{l+1} - 2T_i^{l+1} + T_{i+1}^{l+1}}{(\Delta x)^2}\right] \tag{14}$$

By substituting equations (13) and (14) into the heat equation and collecting term:

$$-\lambda T_{i+1}^{l+1} + 2(1+\lambda)T_i^{l+1} = \lambda T_{i-1}^l + \lambda T_{i+1}^l \tag{15}$$

Such that $\lambda = \frac{k\Delta t}{(\Delta x)^2}$

From equation (15), we derive the first and last interior nodes. Such that for the first interior node

$$2(1+\lambda)T_1^{l+1} - \lambda T_2^{l+1} = \lambda f_0(t^l) + 2(1-\lambda)T_i^l + \lambda T_2^1 + \lambda f_0(t^{l+1}) \tag{16}$$

The last interior node

$$-\lambda T_{m-1}^{l+1} + 2(1+\lambda)T_m^{l+1} = \lambda f_{m+1}(t^l) + 2(1-\lambda)T_m^l + \lambda T_{m-1}^1 + \lambda f_{m+1}(t^{l+1}) \tag{17}$$

Modelling Experiment

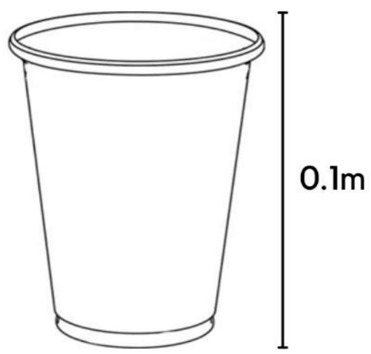

Figure 4. Modelling of experiment

This model experiment will be conducted on ceramic coffee cups. Consider it to be a cylindrical coffee cup with a height of 0.1m. The thermal diffusivity value for ceramic is 1.5×10^{-6} $m^2/$. Let's assume the cup is initially filled with coffee at a temperature of 80°C. This research will be begun on that situation and conducted over a period of

2 hours until the cup's temperature reaches 25°C. The parameters that will be used are time step, $\Delta t = 60s$ and spatial step, $\Delta x = 0.02m$. According to the given details,

$r = \frac{1.5\times10^{-6}}{(0.02)^2} = 0.225$

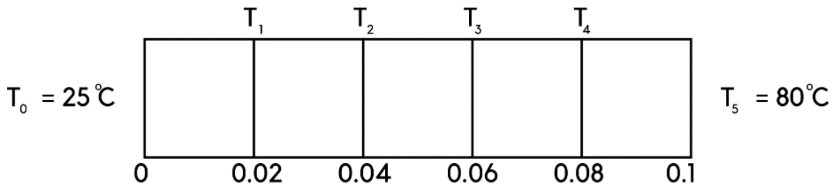

Figure 5. Interior Grid Modelling of the experiment

The boundary conditions can be expressed as below:

$u(t,n) \Rightarrow u(t,0) = 25^0C$, $u(t,0.1) = 80^0C$ and $u(0,n) = 80^0C$
These conditions specify the temperatures at the boundaries and the initial state of the system:

1. **Boundary Condition** $\boldsymbol{x = 0}$: $\boldsymbol{u(t,0) = 25}$℃
- This implies that the temperature at one end of the material remains constant at 25°C for all times t.

2. **Boundary Condition at** $\boldsymbol{x = 0.1}$: $\boldsymbol{u(t,0.1) = 80}$℃
- This indicates that the temperature at the other end of the material is maintained at 80°C for all times t.

3. **Initial Condition**: $\boldsymbol{u(0,x) = 80}$℃
- This denotes that at the initial time $\boldsymbol{t = 0}$, the temperature throughout the material is uniformly 80°C.

Graphical Results

The FTCS Method

The temperature profile inside the coffee cup is shown graphically at various time stages in the results of FTCS simulations. A brief comment on the FTCS graphical results is provided below:

The temperature distribution inside the coffee cup changes over time, as shown by the FTCS simulations' graphical results. The temperature profile is typically plotted at different time intervals over the spatial domain (such as the cup's height or radius). The graphic first displays the temperature distribution that corresponds to the starting circumstances. As the duration increases, the temperature profiles show how heat moves throughout the cup, impacting temperature gradients and the thermal behavior as a whole. These graphical data offer insightful information on the dynamics of heat transport and can be used to comprehend the relationship between many factors (see Figures 6 to 13 and Table 2 to 5).

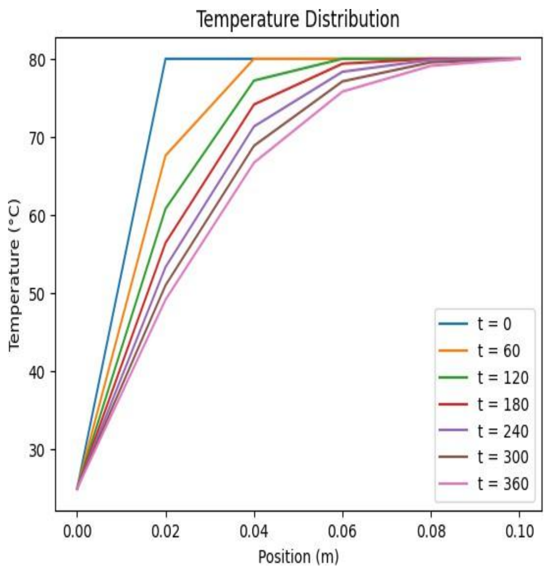

Figure 6. Temperature Distribution of Ceramic

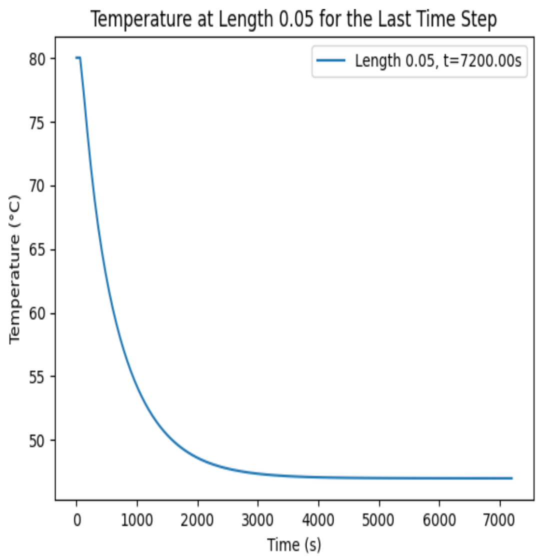

Figure 7. Temperature at the middle length of Ceramic

Table 2. Temperature Distribution of Ceramic

	t\x	x=0.00	x=0.02	x=0.04	x=0.06	x=0.08	x=0.10
0	t=0.00	25.00	80.00	80.00	80.00	80.00	80.00
1	t=60.00	25.00	67.62	80.00	80.00	80.00	80.00
2	t=120.00	25.00	60.82	77.22	80.00	80.00	80.00
3	t=180.00	25.00	56.45	74.15	79.37	80.00	80.00
4	t=240.00	25.00	53.36	71.34	78.34	79.86	80.00
...	...	...	...	...	...	...	...
116	t=6960.00	25.00	36.00	47.00	58.00	69.00	80.00
117	t=7020.00	25.00	36.00	47.00	58.00	69.00	80.00
118	t=7080.00	25.00	36.00	47.00	58.00	69.00	80.00
119	t=7140.00	25.00	36.00	47.00	58.00	69.00	80.00
120	t=7200.00	25.00	36.00	47.00	58.00	69.00	80.00

Numerical Solutions of FTCS method of Thermal diffusivity Ceramic obtained for $t = 7200$ and $\Delta x = 0.02$

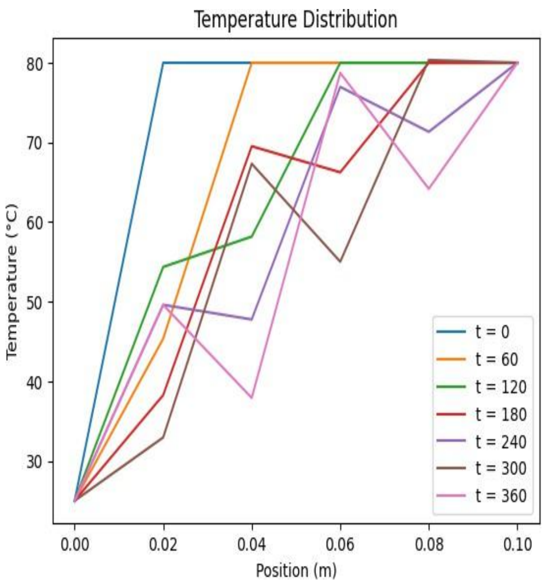

Figure 8. Temperature Distribution of Stainless Steel

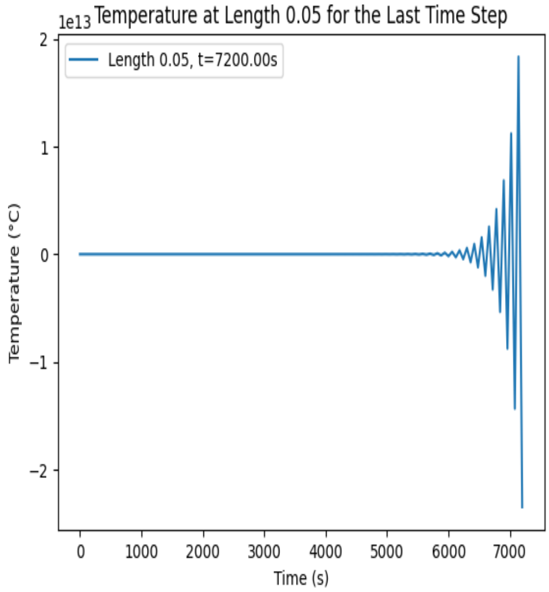

Figure 9. Temperature at the middle length of Stainless Steel

Table 3. Temperature Distribution of Stainless Steel

		x=0.00	x=0.02	x=0.04	x=0.06	x=0.08	x=0.10
0	t=0.00	25.00	80.00	80.00	80.00	80.00	80.00
1	t=60.00	25.00	45.35	80.00	80.00	80.00	80.00
2	t=120.00	25.00	54.36	58.17	80.00	80.00	80.00
3	t=180.00	25.00	38.26	69.52	66.25	80.00	80.00
4	t=240.00	25.00	49.60	47.77	76.97	71.34	80.00
...	...	...	...	...	...	...	...
116	t=6960.00	25.00	5415070883568.79	-8761768740998.98	8761768741103.98	-5415070883463.79	80.00
117	t=7020.00	25.00	-6927832736541.49	11209468836203.58	-11209468836098.58	6927832736646.50	80.00
118	t=7080.00	25.00	8863201878324.79	-14340961888176.18	14340961888281.18	-8863201878219.79	80.00
119	t=7140.00	25.00	-11339238477899.69	18347273263887.57	-18347273263782.57	11339238478004.69	80.00
120	t=7200.00	25.00	14506984160518.84	-23472793445870.59	23472793445975.59	-14506984160413.84	80.00

Numerical Solutions of FTCS method of Thermal diffusivity Stainless-still obtained for $t = 7200$ and $\Delta x = 0.02$

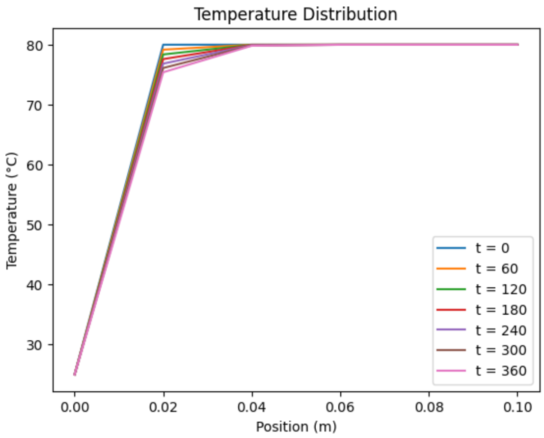

Figure 10. Temperature Distribution of Plastic

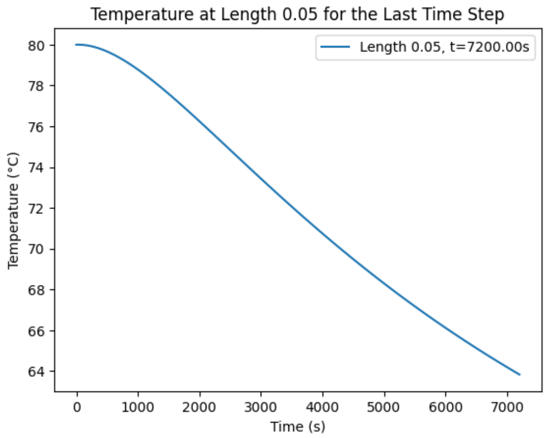

Figure 11. Temperature at the middle length of Plastic

Table 4. Temperature Distribution of Plastic

	t\x	x=0.00	x=0.02	x=0.04	x=0.06	x=0.08	x=0.10
0	t=0.00	25.00	80.00	80.00	80.00	80.00	80.00
1	t=60.00	25.00	79.17	80.00	80.00	80.00	80.00
2	t=120.00	25.00	78.37	79.99	80.00	80.00	80.00
3	t=180.00	25.00	77.60	79.96	80.00	80.00	80.00
4	t=240.00	25.00	76.84	79.93	80.00	80.00	80.00
...	...	...	...	...	...	...	...
116	t=6960.00	25.00	47.56	64.27	73.71	78.02	80.00
117	t=7020.00	25.00	47.47	64.16	73.63	77.98	80.00
118	t=7080.00	25.00	47.39	64.05	73.56	77.95	80.00
119	t=7140.00	25.00	47.30	63.94	73.48	77.91	80.00
120	t=7200.00	25.00	47.22	63.84	73.40	77.88	80.00

Numerical Solutions of FTCS method of Thermal diffusivity Plastic obtained for $t = 7200$ and $\Delta x = 0.02$

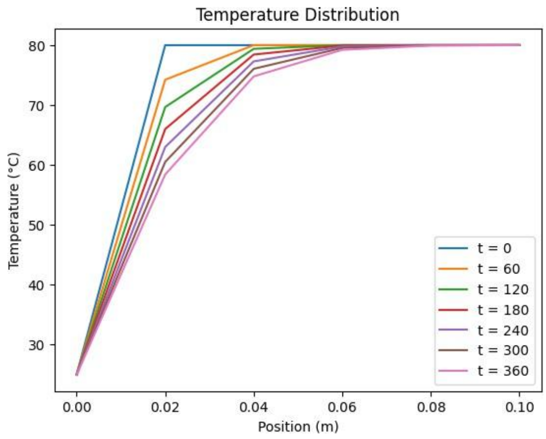

Figure 12. Temperature Distribution of Glass

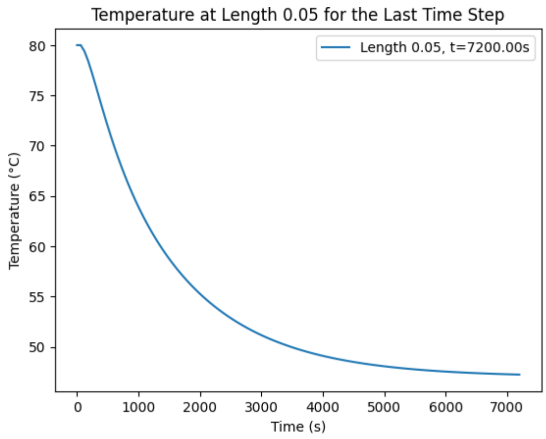

Figure 13. Temperature at the middle length of Glass

Table 5. Temperature Distribution of Glass

	t\x	x=0.00	x=0.02	x=0.04	x=0.06	x=0.08	x=0.10
0	t=0.00	25.00	80.00	80.00	80.00	80.00	80.00
1	t=60.00	25.00	74.22	80.00	80.00	80.00	80.00
2	t=120.00	25.00	69.66	79.39	80.00	80.00	80.00
3	t=180.00	25.00	65.99	78.44	79.94	80.00	80.00
4	t=240.00	25.00	63.00	77.29	79.79	79.99	80.00
...	...	...	...	...	...	...	...
116	t=6960.00	25.00	36.17	47.28	58.28	69.17	80.00
117	t=7020.00	25.00	36.17	47.27	58.27	69.17	80.00
118	t=7080.00	25.00	36.16	47.26	58.26	69.16	80.00
119	t=7140.00	25.00	36.15	47.25	58.25	69.15	80.00
120	t=7200.00	25.00	36.15	47.24	58.24	69.15	80.00

Numerical Solutions of FTCS method of Thermal diffusivity Glass obtained for $t = 7200\ and\ \Delta x = 0.02$

Crank-Nicolson

The crank-Nicolson method's graphical results illustrate temperature distribution within a coffee cup over time, with heightened accuracy compared to FTCS. By incorporating both forward and backward time steps, it offers smoother temperature profiles. This method enables a comprehensive understanding of heat transfer dynamics, including the influence of cup materials like ceramic, glass, stainless steel and plastic on thermal performance. The graphical representation captures evolving temperature gradients, aiding in optimizing coffee cup design for enhanced heat retention and overall thermal efficiency (see Figures 14 to 21 and Tables 6 to 9).

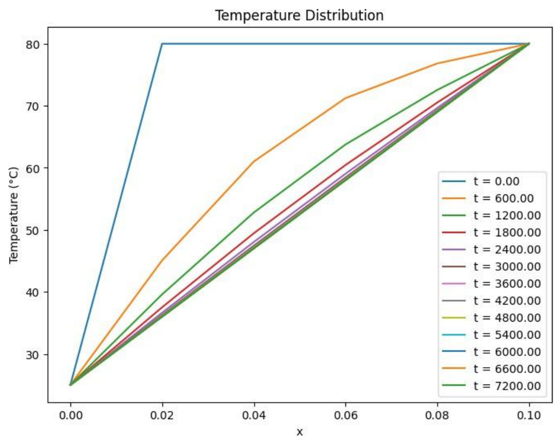

Figure 14. Temperature Distribution of Ceramic

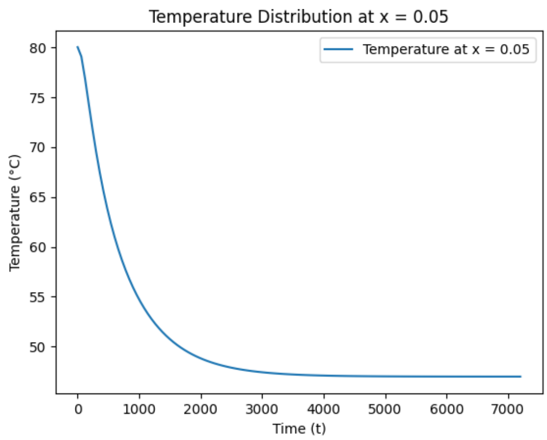

Figure 15. Temperature at the middle length of Ceramic

Table 6. Temperature Distribution of Ceramic

		x=0.00	x=0.02	x=0.04	x=0.06	x=0.08	x=0.10
0	t=0.00	25.00	80.00	80.00	80.00	80.00	80.00
1	t=60.00	25.00	57.88	75.55	79.11	79.83	80.00
2	t=120.00	25.00	50.58	68.17	76.45	79.10	80.00
3	t=180.00	25.00	46.52	63.13	73.04	77.76	80.00
4	t=240.00	25.00	43.91	59.45	70.03	76.22	80.00
...	...	...	...	...	...	...	...
116	t=6960.00	25.00	36.00	47.00	58.00	69.00	80.00
117	t=7020.00	25.00	36.00	47.00	58.00	69.00	80.00
118	t=7080.00	25.00	36.00	47.00	58.00	69.00	80.00
119	t=7140.00	25.00	36.00	47.00	58.00	69.00	80.00
120	t=7200.00	25.00	36.00	47.00	58.00	69.00	80.00

Numerical Solutions of Crank-Nicolson method of Thermal diffusivity Ceramic obtained for $t = 7200$ and $\Delta x = 0.02$

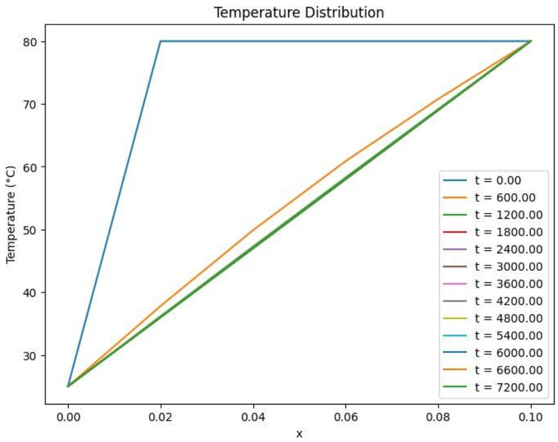

Figure 16. Temperature Distribution of Stainless Steel

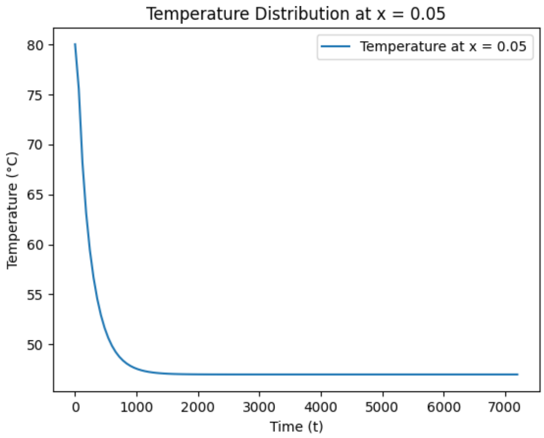

Figure 17. Temperature at the middle length of Stainless Steel

Table 7. Temperature Distribution of Stainless Steel

		x=0.00	x=0.02	x=0.04	x=0.06	x=0.08	x=0.10
0	t=0.00	25.00	80.00	80.00	80.00	80.00	80.00
1	t=60.00	25.00	57.88	75.55	79.11	79.83	80.00
2	t=120.00	25.00	50.58	68.17	76.45	79.10	80.00
3	t=180.00	25.00	46.52	63.13	73.04	77.76	80.00
4	t=240.00	25.00	43.91	59.45	70.03	76.22	80.00
...	...	...	...	...	...	...	...
116	t=6960.00	25.00	36.00	47.00	58.00	69.00	80.00
117	t=7020.00	25.00	36.00	47.00	58.00	69.00	80.00
118	t=7080.00	25.00	36.00	47.00	58.00	69.00	80.00
119	t=7140.00	25.00	36.00	47.00	58.00	69.00	80.00
120	t=7200.00	25.00	36.00	47.00	58.00	69.00	80.00

Numerical Solutions of Crank-Nicolson method of Thermal diffusivity Stainless- steel obtained for $t = 7200\ and\ \Delta x = 0.02$

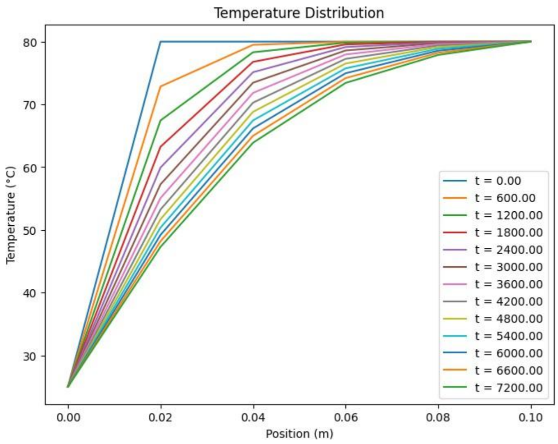

Figure 18. Temperature Distribution of Plastic

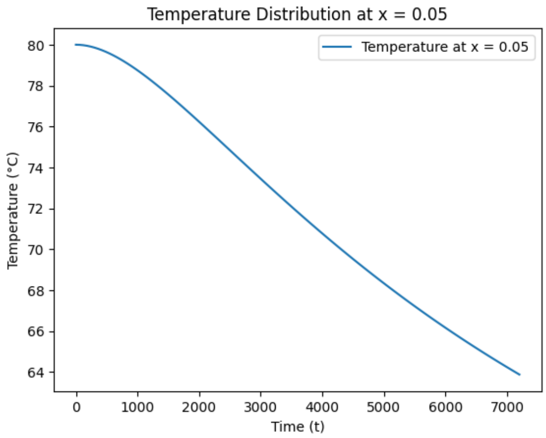

Figure 19. Temperature at the middle length of Plastic

Table 8. Temperature Distribution of Plastic

		x=0.00	x=0.02	x=0.04	x=0.06	x=0.08	x=0.10
0	t=0.00	25.00	80.00	80.00	80.00	80.00	80.00
1	t=60.00	25.00	79.19	79.99	80.00	80.00	80.00
2	t=120.00	25.00	78.40	79.98	80.00	80.00	80.00
3	t=180.00	25.00	77.63	79.95	80.00	80.00	80.00
4	t=240.00	25.00	76.89	79.91	80.00	80.00	80.00
...	...	...	...	...	...	...	...
116	t=6960.00	25.00	47.62	64.31	73.72	78.01	80.00
117	t=7020.00	25.00	47.53	64.20	73.64	77.98	80.00
118	t=7080.00	25.00	47.45	64.10	73.56	77.94	80.00
119	t=7140.00	25.00	47.36	63.99	73.49	77.91	80.00
120	t=7200.00	25.00	47.27	63.88	73.41	77.87	80.00

Numerical Solutions of Crank-Nicolson method of Thermal diffusivity Plastic obtained for $t = 7200$ and $\Delta x = 0.02$

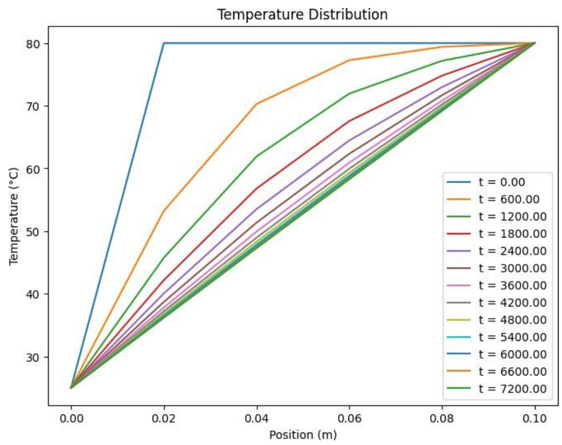

Figure 20. Temperature Distribution of Glass

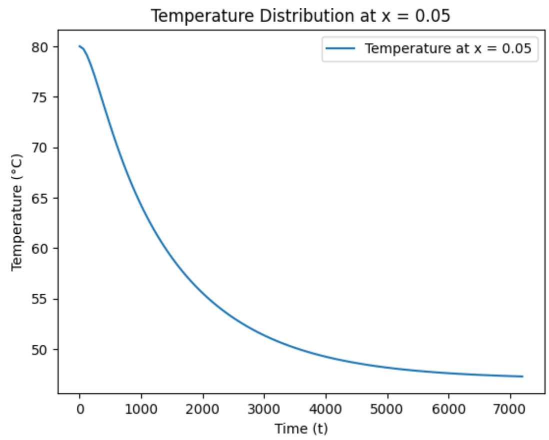

Figure 21. Temperature at the middle length of Glass

Table 9. Temperature Distribution of Glass

		x=0.00	x=0.02	x=0.04	x=0.06	x=0.08	x=0.10
0	t=0.00	25.00	80.00	80.00	80.00	80.00	80.00
1	t=60.00	25.00	74.76	79.75	79.99	80.00	80.00
2	t=120.00	25.00	70.48	79.09	79.94	80.00	80.00
3	t=180.00	25.00	66.93	78.18	79.82	79.98	80.00
4	t=240.00	25.00	63.96	77.12	79.63	79.96	80.00
...	...	...	...	...	...	...	...
116	t=6960.00	25.00	36.19	47.31	58.31	69.19	80.00
117	t=7020.00	25.00	36.18	47.29	58.29	69.18	80.00
118	t=7080.00	25.00	36.18	47.28	58.28	69.18	80.00
119	t=7140.00	25.00	36.17	47.27	58.27	69.17	80.00
120	t=7200.00	25.00	36.16	47.26	58.26	69.16	80.00

Numerical Solutions of Crank-Nicolson method of Thermal diffusivity Glass obtained for t=7200 and Δx=0.02

DISCUSSION

An instability in the numerical solution is revealed by the graph displaying the outcomes of the Forward-Time Central-Space (FTCS) approach. The reason for this instability is that the time step and spatial step sizes selected did not meet the method's stability requirements. The stability of the FTCS method is highly dependent on these factors; if these parameters don't meet the stability requirements of the method, the result becomes erroneous and unreliable. The FTCS method's observed instability highlights how crucial it is to choose the time step size, spatial step size, and diffusion coefficient with care in order to provide accurate and stable results. Numerical instability may result from inadequate consideration of these parameters, jeopardising the solution's fidelity.

To address the instability encountered with the FTCS method, the Crank-Nicolson method was used to overcome the instability that arises with the FTCS method. In contrast to the FTCS approach, the Crank-Nicolson method yields more accurate and dependable solutions for complex dynamics problems and has better stability features, as shown in the numerical and graphical solution From a computational standpoint, the FTCS approach was simpler, but for some materials, it needed more iterations and smaller grids to produce consistent results. Comparing this to the Crank-Nicolson technique, which proved more efficient for bigger time steps and coarser grids despite being implicit and requiring the solution of a system of equations at each time step, resulted in an increase in the total computing time. The Python implementation brought to light the real-world difficulties and optimisations required for each technique, such as using sparse matrix solvers to increase the effectiveness of the Crank-Nicolson approach. To summarise, the instability noted in the FTCS approach emphasises the necessity of careful parameter selection and evaluation of alternative numerical techniques in order to guarantee the accuracy and robustness of numerical solutions in heat transfer issues.

CONCLUSION

This study investigates the Initial Boundary Value Problem (IBVP) associated with the one-dimensional heat equation, focusing on four distinct materials: ceramic, stainless steel, plastic, and glass. The aim is to analyze their thermal diffusivity characteristics using numerical methods, namely the explicit FTCS method and the Crank-Nicolson method. By formulating the heat equation and defining appropriate boundary conditions, we simulate the thermal behaviour of the materials under consideration. Python codes are implemented with uniform step sizes to facilitate comparative testing across the materials. Our results reveal notable disparities in thermal diffusivity performance among the materials, with stainless steel emerging as the most efficient conductor due to its superior thermal diffusivity. Furthermore, a detailed examination of cup temperature dynamics at the midpoint provides insights into the interplay between time and temperature. Comparative analysis between the explicit FTCS and Crank-Nicolson methods demonstrates the latter's advantage in providing more accurate and stable approximations. The unconditional stability of the implicit Crank-Nicolson scheme contrasts with the conditional instability of the explicit FTCS method, underscoring the importance of method selection in numerical simulations.

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