PRE-UNIVERSITY STUDENTS' DIFFICULTIES IN PROBLEM SOLVING BY USING Q-METHODOLOGY

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Abstract

Problem solving is very important in learning mathematics. During mathematical problem solving, all the mathematical concepts and skills are important particularly in their applications as well as in making decisions. However, students were reported to have difficulties in mathematics problem solving eventhough they were in the post-secondary education level. This level is less researched compared to those of the primary and secondary education levels. The focus of this paper is to present the students' viewpoints on their difficulties in solving mathematics problems from cognitive ability aspect. The study was carried out using Q-Methodology which is a systematic study of subjectivity in order to have a clearer understanding in studying subjective viewpoints. Data were analyzed using factor analysis of the PQMethod Program. Findings showed the respondents lacked in mathematical cognition such as resource, heuristic and control. The understanding on how these factors influence problem solving is expected to give effective guidelines in preparing diagnostic instruments and learning modules by teachers in order to develop students' mathematics skills.

Keywords *Mathematics problem solving, Mathematics difficulties, Q-methodology, PQMethod 2.33.*

Abstrak

Penyelesaian masalah amatlah penting dalam pembelajaran matematik. Semasa menyelesaikan masalah matematik, semua konsep dan kemahiran matematik adalah penting terutama sekali dalam aplikasi dan juga dalam membuat keputusan. Walau bagaimanapun, pelajar didapati mempunyai kesukaran dalam menyelesaikan masalah matematik walaupun berada di peringkat lepasan menengah. Tidak banyak kajian dilakukan di peringkat ini berbanding dengan peringkat menengah dan rendah. Tumpuan kertas kerja ini adalah untuk membentangkan dapatan yang diperoleh tentang pandangan pelajar berkaitan kesukaran menyelesaikan masalah matematik dari segi kebolehan kognitif. Kajian ini mengguna kaedah *Q-Methodology* yang merupakan kaedah kajian sistematik tentang subjektiviti, untuk memahami dengan lebih jelas dalam mengkaji pandangan subjektif. Dapatan data dianalisis menggunakan faktor analisis dari Program PQMethod. Dapatan menunjukkan terdapat kelemahan pelajar dalam aspek kognitif matematik iaitu sumber, heuristik, kawalan kendiri (metakognitif). Pemahaman tentang

bagaimana faktor ini mempengaruhi penyelesaian masalah matematik dijangka memberi panduan yang berkesan kepada guru apabila menyediakan instrumen diagnostik dan modul pembelajaran untuk membina kemahiran matematik pelajar.

Kata kunci *Penyelesaian masalah matematik, kesukaran matematik, kaedah Q-methodology, PQMethod 2.33.*

INTRODUCTION

Mathematics problem solving can be easily solved by some students but not to some others. Understanding the specific difficulties might lead to a more meaningful teaching and learning process. There are so many reasons for students' abstruse in learning mathematics. Some studies showed students have difficulties in understanding and retrieving concepts, formulae, facts and procedures (Zahrah et al., 2003). In a study on "Students' Difficulties in Mathematics Problem-Solving: What do they say" by Tambychik and Meerah (2010), it was found from the students' points of view, major mathematics skills and cognitive abilities in learning were the causes of the difficulties in mathematics problem solving. The subjects were 14 year old students. From the study, it was reported, the students'cognitive ability to concentrate and to recall was poor. While in the mathematics skills, the major sub skills of ensuring accurate and systematic working procedures were the main causes of the difficulties in problem solving.

The Mathematics curriculum in Malaysia for secondary schools aims to develop students who are able to think mathematically and apply mathematical knowledge in problem solving as well as making decisions. The development of problem solving skills is being emphasized during the teaching and learning process. The skills involved the four main steps in problem solving such as: interpreting the problem, planning the strategy, carrying out the strategy, and reflecting on the solution. These skills are adopted from Polya (1945) who stated there are four problem solving stages. They are; 1) Understanding the problem: identifying the problem's known (given) and unknown and, if appropriate, using suitable notation, such as mathematical symbols to represent the problem, 2) Devising a plan: determining appropriate actions to take to solve the problem, 3) Carrying out the plan: executing the actions that have been determined to solve the problem and checking their effectiveness, 4) Looking backward: evaluating the overall effectiveness of the approach to the problem with the intention of learning on how similar problems may be solved in future occasions.

There are variables suggested by previous researchers to explain the success or failure in solving problems. The variables are knowledge, heuristics, metacognition and beliefs (Schoenfeld, 1985) as well as skills, metaskills and will (Mayer, 1998). Knowledge refers to the resources the students have, heuristics are the strategies or skills to use, metacognition is monitoring and self-regulation while beliefs are the students' view and perceptions of themselves and their mathematics ability as well as motivation (Schoenfeld, 1985; Mayer, 1998). It was found effective cognitive and metacognitive processes and strategies for mathematics problem solving helped

students on how to apply those processes and strategies when solving problems (Krawec et al., 2012). Sharma (2012) stated that factors responsible for mathematics learning are intellectual and perceptual development, mathematics language, levels of knowing in the development of mathematics milestones, as well as the pre-requisites and support skills in mathematics.

Problem Solving Theory and Students' Difficulties

In this study, the problem solving theory by Alan Schoenfeld (1985) was adopted as his theory is generally applicable to college level mathematics. According to this theory, successful mathematics problem solving depends on a combination of resource knowledge, heuristics, control and belief, all of which had be mentioned earlier. This paper however focuses only on the resource, heuristics and control domains to explain students' difficulties in solving mathematical problems.

Matriculation students had moderately favorable attitude towards algebra problem solving with only a small number of students were incapable to understand what a given question meant or what it required (Zakaria & Yusoff, 2009). On the other hand university students were found not able to relate mathematics problems to their existing knowledge. It was also reported, a few students could not strategize plans for solving the problems correctly, could not monitor the problem solving process entirely from the beginning to the end, did not know how to choose methods most suited for solving each problem and not re-checking the results. The majority of the students might only follow rules such as multiplying out of the brackets, collecting together like terms and looking for common factors (Yunus et al., 2006; Tarmizi, 2010).

The performance of mathematics problem solving among university students was shown as poor or average or moderate (Yunus et al., 2006; Bayat & Tarmizi, 2010). However, there was a positive, but moderate significant relationship between algebra problem solving performance, mathematical achievement and meta-cognitive strategies and its subscales (Bayat & Tarmizi, 2010). In a study (Samo, 2009), it was found students' perceptions on symbols, letters and signs in algebra affected their learning of algebra. Other students' perceptions that affected the learning of algebra include the specific unknowns, the use of the generalization of terms, expressions and equations. From Schoenfeld (1992), there were five aspects of cognition needed in solving mathematical problem; 1) the knowledge base, 2) problem solving strategies, 3) monitoring and control, 4) beliefs and affects, and 5) practices. Difficulties in either one of these aspects would cause difficulties in problem solving. In Ong and Lim's study (2014), the major difficulties for Penang Matriculation's students in problem solving were understanding and interpreting the symbolic notations used in algebra. Other difficulties were lack of understanding of the questions, failing to transform the questions into mathematical symbols, the uncertainty of the methods to be used, and no confidence in the complicated mathematics. According to Ong and Lim (2014), students in Penang Matriculation College have difficulties with mathematical symbol that relect their confidence in answering mathematics problems especially in questions involved of mathematical symbols.The study also revealed students were quite weak

in transforming the information from mathematics questions into the mathematics language. This paper discusses students' points of view on problem solving difficulties from the three aspects of the cognitive domain; knowledge, problem solving strategies (heuristics) and control.

METHODOLOGY

Q-methodology approach was used to gain a clearer understanding of students' difficulties in mathematics problem solving. Q-methodology was introduced by a psychologist/physicist William Stephenson in 1930's (Watts & Stenner, 2005; 2012). Q-methodology provides a foundation for the systematic study of subjectivity, and is recommended for qualitative study on human behavior (Brown, 1993). The basis of Q-methodology is the Q-sort technique, followed by Q factor analysis. Stephenson presented Q-methodology as an inversion of conventional factor analysis; through centroid factor analysis (CFA) as factor extraction technique (Watts & Stenner, 2012). In Q-sample development, the informations based on the concourse statements, are from interviews, articles or collections of paintings, pieces of art, photographs, and even musical selections (Brown, 1993). Typically, a concourse can be located in individual or group interviews, contributions from respondents, or published materials (Davis & Michelle, 2011). All the informations from the concourse statements are used in the development of the instrument for this study, and the Q-sample is a contribution and not part of the results or findings of the study.

The Development of Q-Sample

The concourse for this study was collected through personal interviews with current students, the lecturers, the discussions among the mathematics' educators and from literature reviews. A concourse is the collection of relevant ideas, beliefs and views on the topic of study from a wide range of sources (McKeown & Thomas, 1988). All the informations from the mentioned processes were used as concourse statements to provide the fullest range of viewpoints. No specific number of statements are required in the Q-Sample, however, 40 to 50 statements are adequate, as long as they are comprehensive (Brown, 1980). For this study's concourse, there were 42 card statements, called the Q-set, which became the instrument.

P-set

P-Set or Person Sample in Q-Methodology is a set of individuals who are purposively selected to do the Q-sorting according to their personal attributes, views they may express, or on the basis of their social position and background. That means in Q-Sampling the individuals are purposively selected to be the participants. Participants in a study involving Q-Methodology are not randomly chosen (McKeown & Thomas, 1988). As such, Q studies generally do not need a large sample of participants. Not more than 40 participants are necessary to represent the viewpoints of a population (Brown, 1980). However, Watts and Stenner (2005) stated most Q studies are effective with 40-60 participants but this is merely a guideline. The objective of this paper was to view the students' perceptions on their difficulties in problem-solving. Thirty

one (31) accounting students from Negeri Sembilan Matriculation College willingly participated and be part of the P-Set.

Q-sorting

Q-sorting is a data gathering process in Q study where Q-set is the instrument. During the sorting procedure, each respondent has to create a model of his/her viewpoints using the items in the Q-set. Q-set items or statements are transferred onto separate cards, randomized and numbered. The participants are required to sort 42 statements into a "forced-choice" Normal Distribution with preliminary sorting into three categories of agree, disagree, or neutral. In this study, the rating scale was spread across the top of a flat area that ranged from -4 to +4 as shown in Figure 1.

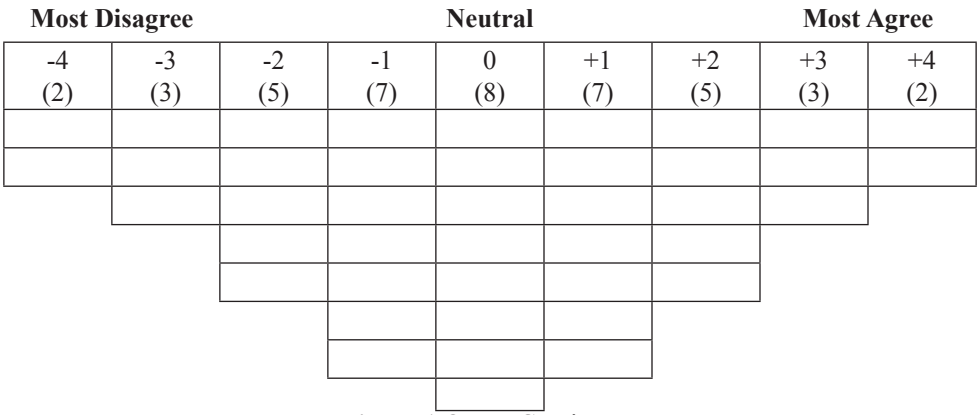

Figure 1 Q-sort Continuum

FINDINGS AND DISCUSSION

Data analysis in Q-methodology typically involves the sequential application of three sets of statistical procedures: correlation, factor analysis, and the computation of factor scores. The first output analysis from the software is the correlation matrix. This correlation matrix shows the similarity or dissimilarity between participants. Correlations range from -1 to 1, a negative correlation indicating that Q sorters have ranked the items differently. In this study, a proprietary software package by PQMethod 2.33 was used to analyse the grids. The unrotated factors matrix demonstrated the eigenvalue criteria of the possible 8 factors had eigenvalues of more than 1.00 which were considered significant with 74% variance (high percentage). The Eigenvalues reflected the amount of variations accounted for by the corresponding factor. In essence, the relative magnitude of the Eigenvalues could be used to rank the importance of the factors. After the varimax rotation, there were 4 selected factors being rotated with 52% variance. The ranking showed Factor 1 which was on problem solving strategies (heuristics) showed 17% ; Factor 2 which was on basic concept of knowledge, showed 15%; Factor 3 which was on control, showed 10%; while Factor 4 which was again on problem solving strategies (heuristics) showed 10%. Tables 1, 2, 3 and 4 present the highest ranked and lowest ranked of the factors.

Table 1 Highest Ranked and Lowest Ranked Statements:Factor 1

Q-sort	Statements	Rank	Z-Score
	Agreement Statements		
32	I have difficulties in using backward strategy in my solution.	+4	2.21*
29	I have difficulties in solving equations involving 3 surds.	+2	0.86
21	I have difficulties in recalling a similar problem that I had worked before.	+2	0.75
	Disagreement Statements		
39	I have difficulties in solving linear inequalities.	-2	-0.93*
41	I have difficulties in solving the multiplication of two expressions.	-2	-0.98
10	I have difficulties in solving the multiplication of fractions as a single fraction such as $11 \times \frac{x}{3} = \frac{11 \times x}{3} = \frac{11x}{3} = \frac{11}{3}x$ $11 \times \frac{x}{3} = \frac{11 \times x}{3} = \frac{11x}{3} = \frac{11}{3}x$.	-3	-1.52*
15	I have difficulties in solving algebraic expressions using associative, commutative and distributive laws such as $a(x-y) = ax + ay$ $a(x-y) = ax + ay$ and $a(xy) = axy$ $a(xy) = axy$.	-3	-1.66*
24	I have difficulties in understanding the meaning of 3^n which is a repeated multiplication of $3 \times 3 \times 3 \ldots \ldots \times 3$ $3 \times 3 \times 3 \ldots \ldots \times 3$ for *n* times.	-4	-1.77*
19	I have difficulties in solving the multiplication of fractions as a single fraction such as $4 \times \frac{2}{x} = \frac{2}{x} + \frac{2}{x} + \frac{2}{x} + \frac{2}{x} = \frac{8}{x}$ $11 \times \frac{x}{3} = \frac{11 \times x}{3} = \frac{11x}{3} = \frac{11}{3}x$.	-4	-2.19*

Factor 1 identified the most learning difficulties in problem solving from the participants' view through the statements such as; "I have difficulties in using backward strategy in my solution". It showed heuristic difficulties in mathematical problem solving such as the *solving backwards* strategy. However, the paticipants marked -4 (disagree) for statements such as "I have difficulties in understanding the meaning of 3^n which was a repeated multiplication $3 \times 3 \times 3 \ldots \ldots \times 33 \times 3 \times 3 \ldots \ldots \times 3$ for *n* times" and "I have difficulties in solving multiplication of fractions as a single fraction such as $4 \times \frac{2}{x} = \frac{2}{x} + \frac{2}{x} + \frac{2}{x} + \frac{2}{x} = \frac{8}{x} 4 \times \frac{2}{x} = \frac{2}{x} + \frac{2}{x} + \frac{2}{x} + \frac{2}{x} = \frac{8}{x}$". It showed the participants had no difficulties in the multiplication of fractions and the meaning of an index number.

Table 2 Highest Ranked and Lowest Ranked Statements:Factor 2

Q-sort	Statements	Rank	Z-Score
	Agreement Statements		
19	I have difficulties in solving the multiplication of fractions as a single fraction such as $11 \times \frac{x}{3} = \frac{11 \times x}{3} = \frac{11x}{3} = \frac{11}{3}x$ $4 \times \frac{2}{x} = \frac{2}{x} + \frac{2}{x} + \frac{2}{x} + \frac{2}{x} = \frac{8}{x}$.	+4	2.16*
10	I have difficulties in solving the multiplication of fractions as a single fraction such as $11 \times \frac{x}{3} = \frac{11 \times x}{3} = \frac{11x}{3} = \frac{11}{3}x$ $11 \times \frac{x}{3} = \frac{11 \times x}{3} = \frac{11x}{3} = \frac{11}{3}x$.	+4	1.82
16	I have difficulties in using my mathematical knowledge while doing problem solving.	+2	0.92
9	I have difficulties in solving index number problem because of index laws.	+2	0.88
35	I have difficulties in using the definition of absolute values.	+2	0.83*
	Disagreement Statements		
25	I have difficulties in solving simultaneous equations involving 3 unknown.	-3	-1.42
41	I have difficulties in solving the multiplication of two expressions.	-4	-1.86*

Factor 2 identified the most learning difficulties in problem solving from the participants' view through the statements such as; "I have difficulties in solving the multiplication of fractions as a single fraction such as $11 \times \frac{x}{3} = \frac{11 \times x}{3} = \frac{11x}{3} = \frac{11}{3}x$ $11 \times \frac{x}{3} = \frac{11 \times x}{3} = \frac{11x}{3} = \frac{11}{3}x$ " and "I have difficulties in solving the multiplication of fractions as a repeated addition such as $4 \times \frac{2}{x} = \frac{2}{x} + \frac{2}{x} + \frac{2}{x} + \frac{2}{x} = \frac{8}{x}$ $4 \times \frac{2}{x} = \frac{2}{x} + \frac{2}{x} + \frac{2}{x} + \frac{2}{x} = \frac{8}{x}$". The statements were on the knowledge factor with basic concepts of algebra and fractions. Factor 2 showed the participants' difficulties in knowledge base algebra involving fractions. However, there were no difficulties in solving simultaneous equations as well as in solving the multiplication of two expressions.

Table 3 Highest Ranked and Lowest Ranked Statements: Factor 3

Q-sort	Statements	Rank	Z-Score
	Agreement Statements		
6	I have difficulties in solving equations involving 2 surds.	+4	2.11*
9	I have difficulties in solving index number problem because of index laws.	+4	1.61
29	I have difficulties in solving equations involving 3 surds.	+3	1.59
40	I have difficulties in solving equations of logarithm with different log base.	+3	1.45*
2	I have difficulties in multiplying two surds.	+3	1.42*
	Disagreement Statements		
30	I have difficulties in identifying the useful information that was given in the problem.	-3	-1.42*
20	I have difficulties in applying mathematics I learnt in other subject.	-3	-1.42*
22	I have difficulties in applying the alternative strategy in solving problems.	-3	-1.62*
23	I have difficulties in identifying my mistakes during problem solving.	-4	-1.89*
5	I have difficulties in understanding the information in problem solving.	-4	-1.90*

From Table 3, Factor 3 identified the most learning difficulties in problem solving from the participants' view through the statements such as; "I have difficulties in solving equations involving 2 surds" and "I have difficulties in solving index number problem because of index laws". These were statements which showed difficulties in the knowledge of algebra. However, in Factor 3, the participants did not seem to have difficulties in the heuristics domain such as understanding the information, identifying mistakes, applying alternative strategy and mathematics concept, and identifying useful information.

From Table 4, Factor 4 identified the most learning difficulties in problem-solving from the participants' view, through the statements such as; "I have difficulties in using teacher's solution for similar problem to do problem solving" and "I have difficulties in differentiating the union and the interception of intervals on number line" and "I have difficulties in applying the correct strategy for solving problem". The respondents did not see "I have difficulties in applying table of sign/graph/number line to solve quadratic inequalities" as difficulties in learning mathematics.

Table 4 Highest Ranked and Lowest Ranked Statements: Factor 4

Q-sort	Statements	Rank	Z-Score
	Agreement Statements		
12	I have difficulties in using teacher's solution for similar problems in problem solving.	+3	1.64*
34	I have difficulties in differentiating the union and the interception of intervals on number line.	+3	1.60*
18	I have difficulties in applying the correct strategy for solving problems.	+3	1.48*
	Disagreement Statements		
3	I seldom draw diagrams or pictures to help me understand the problems.	-3	-1.37
8	I have difficulties in solving equations of index numbers.	-3	-1.38
37	I have difficulties in applying the table of signs/graphs/ number lines to solve quadratic inequalities.	-4	-2.19*

The findings showed consistencies with some studies such as those reported by Ong and Lim (2014), Phonapichat et al., (2014), Nadirah et al., (2012), Chow (2011), Zakaria and Yusoff (2009), Yeo (2009), and Yong and Kiong (2005). There were difficulties among the participants in understanding mathematical symbols, solving algebraic problems, understanding the problems, choosing wrong strategies in problem solving and in understanding basic algebra concepts.

For this study, from Factor 1, Factor 2, Factor 3 and Factor 4, in the domain of *resources*, students showed to have difficulties in solving equations involving 2 and 3 surds, difficulties in solving multiplication of fractions as a single fraction such as $4 \times \frac{2}{x} = \frac{2}{x} + \frac{2}{x} + \frac{2}{x} + \frac{2}{x} = \frac{8}{x}$ and $11 \times \frac{x}{3} = \frac{11 \times x}{3} = \frac{11x}{3} = \frac{11}{3}x$, difficulties in solving index number problem because of index law, difficulties in using definition of absolute values, difficulties in solving equations of logarithm with different log base, difficulties in multiplying two surds, and difficulties in differentiating the union and the interception of intervals on number line. While in the domain of heuristic, students showed to have difficulties in using *backward* strategy, difficulties in using their mathematical knowledge while doing problem solving, and difficulties in applying the correct strategy for solving problem. Ultimately in domain of *control*, the students have difficulties in recalling on a similar problem that they had worked before and also in using teacher's solution for a similar problem.

CONCLUSION

This paper shows that participants lacked mathematical cognition such as resource, heuristics and control. The analysis revealed four dominant factors in learning difficulties. Those were; Factor 1 which was on problem solving strategies/ heuristics,

Factor 2 was on basic concepts of knowledge , Factor 3 was on the control factor, while factor 4 was again on problem solving strategies / heuristics.

Schoenfeld's Theory on problem solving was adopted for this level of education. It showed pre-university students' perceptions in their problem solving difficulties in factors such as knowledge, heuristics and control. As Q Methodology is a tool for uncovering perceptions and not a tool for isolating direct cause and effect, hence this study could be used by teachers to identify the difficulties of the students in solving mathematical problems. Much attention should be directed to fostering students' ability to do problem solving. Students' viewpoints are a very useful tool and a very clear guide to teachers in helping the students with their individual difficulties in problem solving.

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