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An Analysis of Students' Understanding of Algebraic Concepts

Analisa Kefahaman Pelajar terhadap Konsep Algebra

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ABSTRACT

Teaching algebraic topics needs more exceptional efforts to make connections among the topics. Prior research has emphasised students' accomplishments in specific subject areas, but there is a deficiency in analysing students' skills in a few related subject areas. This study seeks to determine the extent to which students comprehend the relationship between the topics. A paper-and-pencil examination was administered to 102 pupils from two private secondary schools in Malaysia using a case study. Five respondents were selected to be interviewed regarding their understanding of these topics. This study revealed the efforts made by students to improve their understanding of algebra. A comparison was made between their algebraic knowledge as it pertained to quadratic equations, inequalities, and graphs. Their limited understanding of graphical knowledge in comparison to quadratic equations and inequations influences the decision of the necessary steps for the fundamental components of quadratic mathematics. As there are connections between the topics, this study suggests that students can bridge gaps between their current algebraic knowledge and their graphic comprehension of quadratic functions, equations, and inequalities through discourse and interactive activities.

Keywords: algebra, quadratic equations, quadratic inequalities, quadratic graphs, students' understanding.

ABSTRAK

Mengajar topik algebra memerlukan usaha yang lebih untuk membuat perkaitan antara topik. Kajian terdahulu telah menekankan pencapaian pelajar dalam bidang mata pelajaran tertentu, tetapi terdapat kekurangan dalam menganalisa kemahiran pelajar dalam beberapa bidang mata pelajaran yang berkaitan. Kajian ini bertujuan untuk menentukan sejauh mana pelajar memahami perkaitan antara topik. Ujian berbentuk kertas dan pensel telah diberikan kepada 102 pelajar dari dua sekolah menengah swasta di Malaysia bagi menjalankan kajian kes ini. Lima orang responden telah dipilih untuk ditemu bual berkenaan kefahaman mereka terhadap tajuk-tajuk tersebut. Kajian ini mendedahkan usaha yang dilakukan oleh pelajar untuk meningkatkan pemahaman mereka tentang algebra. Perbandingan telah dijalankan antara pengetahuan algebra berkenaan, iaitu perkaitan pengethauan antara persamaan kuadratik, ketaksamaan dan graf. Pemahaman mereka yang terhad tentang pengetahuan grafik berbanding dengan persamaan dan ketaksamaan kuadratik mempengaruhi penentuan langkah-langkah yang diperlukan untuk komponen asas matematik kuadratik. Oleh kerana terdapat perkaitan antara topik, kajian ini mencadangkan bahawa pelajar boleh merapatkan jurang antara pengetahuan algebra semasa mereka dan pemahaman grafik mereka tentang fungsi kuadratik, persamaan, dan ketaksamaan melalui perbincangan dan aktiviti interaktif.

Kata kunci: algebra, persamaan kuadratik, ketaksamaan kuadratik, graf kuadratik, pemahaman pelajar

INTRODUCTION

Proficiency in mathematics necessitates the ability to discern and contrast concepts effectively. Particularly when numerous mathematical processes, such as establishing connections, are involved, these skills are vital (García-García & Dolores-Flores, 2021; Rodríguez-Nieto et al., 2022). Empirical data have identified three fundamental types of comparisons: (1) problem-to-problem comparisons, (2) step-to-step comparisons, and (3) item-to-item comparisons (Hattikudur & Alibali, 2011). The use of compare-and-contrast strategies is of paramount importance in solving mathematical problems (Jacobs Danan & Gelman, 2018). The importance of this becomes notably evident during the fraction curriculum in elementary school, where students compare fractions and decimals, percentages and fractions, and decimals and percentages (Baiduri, 2020; Spitzer *et al*., 2023). Conversely, upon progressing to secondary education, pupils are anticipated to possess a proficient command of algebra. Students are expected to develop comprehension in the comparisons, including quadratic equations, quadratic inequalities, and quadratic graphs, due to the interconnected nature of numerous topics.

When students pursue study at a higher level, algebraic knowledge becomes important. Many mathematical problems require this knowledge, such as higher level thinking in fractions (Adnan *et al.*, 2018) when solutions are carried out (Binkley, 2023). A Basic understanding of algebraic basis in equations, inequalities, and graphs is still catching educators' attention since difficulties in carrying out algebraic calculations still fail among students (Mohd. Tajudin *et al*., 2015). Many tertiary students encounter persistent challenges in grasping algebraic concepts and applying them effectively. Bridging this gap is crucial to their success in STEM disciplines.

A prominent area where algebraic skills are applied is in understanding the concepts of equality and inequality. Equality is commonly understood as a state of equivalence, where two quantities or groups possess identical values. The equal sign (=) in mathematical notation is a symbol for equality. On the other hand, inequality signifies the difference between two quantities or groups, indicating that one is greater than the other. This difference can be expressed using various mathematical symbols, including $>, \geq, <$, and $\leq$. Inequality describes unbalanced or unequal relationships between quantities, values, or groups.

Conversely, the process of comprehension of equation and inequality implies that additional efforts, like the use of comparative representations, are pivotal for achieving proficiency in mathematics (Pape & Tchoshanov, 2001). When it comes to mastering quadratic equations and inequalities, employing graphical and tabular representations proves instrumental in facilitating the comparison of this knowledge.

Typically, students embark on their journey into the world of functions with a broad understanding, gradually progressing to more specialised types, such as linear, quadratic, exponential, and trigonometric functions. This sequential approach enables students to construct a robust foundation in fundamental function concepts, which they can then apply to specific functions like quadratic functions and equations. This method also encourages students to recognise the connections and parallels between functions, equations, and inequalities, fostering a deeper grasp of mathematics. It was informed that functions and quadratic functions have a relationship. Comprehension of function is essential before understanding of quadratic functions of content exploration closely. Therefore, the primary aim of this study is to evaluate students' proficiency in solving equations, inequalities, and effectively employing graphical representations.

Specifically, this study addresses the following research questions:

Research Question 1: What are the students' performances in quadratic equation, quadratic equation inequalities, and quadratic graph?

Research Question 2: What are the students' perceptions towards their learning in the quadratic lessons?

Research Question 3: What are students' misconceptions in answering the quadratic questions?

LITERATURE REVIEW

Quadratic equation instruction and learning have received heightened interest in the digital age due to the increasing reliance of contemporary education on digital platforms and tools to illustrate and communicate mathematical concepts (Poçan et al., 2023). Educators have begun to acknowledge the potential of technology integration in education to augment comprehension and involvement in mathematics, specifically with regard to quadratic equations.

The complexity surrounding the comprehension of inequalities and equations has been well documented. The confusion often prolongs the learning process as more concepts are developed from the differences (Tsamir & Almog, 2001). Tsamir and Almog (2001) found that students commonly struggle to differentiate between inequalities and equations. This difficulty can significantly impede their progress in algebraic understanding, particularly when working with quadratics (O'conor, 2022). It depicts a challenging aspect of algebraic learning—graphing quadratic functions. The proficiency in this skill was identified as the lowest among the assessed topics. This finding aligns with empirical evidence supporting the notion that effectively dealing with graphical representations of mathematical concepts demands a more advanced level of knowledge and skill (Mainali, 2021). The studies emphasise that the strong acquisition with quadratic graphs extends beyond the comprehension of underlying equations; it necessitates the ability to translate that understanding into meaningful graphical representations.

The crucial role that graphical representation plays in enhancing students' algebraic abilities underscores the importance of learning quadratic equations and inequalities. Therefore, graphical representation functions as a powerful instrument for visualising mathematical concepts, establishing a concrete connection between abstract algebraic expressions and practical implementations, particularly in the context of learning graph functions. As guided by Gagné's cognitive learning theory (Gagne, 1985), which emphasises the roles of five taxonomies: the use of verbal information, intellectual skills, motor skills, cognitive skills, and attitude, the integration of graphical representation aligns with the cognitive skills taxonomy. In addition to improving students' decoding of mathematical symbols, active engagement in graphing activities fosters the development of cognitive abilities through the promotion of exploration and manipulation of algebraic concepts. By integrating symbolic and visual aids, this comprehensive methodology is consistent with efforts to develop a lasting comprehension of quadratic mathematics. By promoting active engagement and manipulation of algebraic concepts, this method cultivates within students a more profound and interrelated comprehension of quadratic equations, inequalities, and graphs.

RESEARCH METHODOLOGY

A mixed-method approach was employed to collect quantitative and qualitative data. The population for this study included all urban private secondary schools in Seremban District, located in Negeri Sembilan, Malaysia. Based on a few criteria, including principal approval, availability of additional mathematics subjects, and logistical accessibility, two private schools were chosen to participate in the study. A total of 102 samples from the two schools were randomly selected to answer the instruments. Random sampling was employed with consideration of students' mathematical backgrounds and their willingness; hence, 102 samples participated in this study. Two instruments (adapted from (Grewal, 1994)) were employed in this study, namely the Diagnostic Test and the questionnaire regarding students' attitudes towards general mathematics classroom transactions and learning quadratics topics. A Likert-scale questionnaire with five levels of choices from strongly disagree to strongly agree was prepared for students to answer the 15 items in the questionnaire. Table 1 presents the sample items of the test and questionnaire.

Table 1 : The sample items

Sample of the test	Samples of questionnaire item
If the graph of $f(x) = ax^2 + bx + c$ intersects y-axis it means that $ax^2 + bx + c = 0$ has two distinct real roots. [True or False] Minimum point A of the parabola shown in the graph lies on the intersection of lines $y = 4$ and $x = 2$. Point $C\ (4\ ,5)$ lies on the parabola. Therefore, the defining equation of the parabola is: $y = x^2 - 4x + 8$.	I think mathematics can be learned using the discovery method. I found quadratic inequalities difficult, so I do not spend much time learning this section. I do not know how to do a graphical solution of quadratic equation

FINDINGS AND DISCUSSIONS

Research Question 1: What are the students' performances in quadratic equation, quadratic equation inequalities, and quadratic graph?

The mean score for items involving quadratic equations, quadratic inequalities, and graphs is 57.20% (5.72 out of 10), 60.27% (6.63 out of 11), and 46.31% (6.02 out of 13), with the overall mean 18.36 (out of 34) as shown in Table 2. For many students, learning and comprehending quadratic equations, inequalities, and quadratic graphs are challenging since they require a conceptual understanding of quadratic equations (Joshi, 2019). Besides, confusion between inequalities and equations has taken them more time to comprehend (Tsamir & Almog, 2001). The study's data indicated that the level of comprehension and proficiency in graphing quadratic functions was the lowest among the assessed topics. This finding is supported by empirical evidence, which suggests that effectively dealing with graphical representations of mathematical concepts demands a more advanced level of knowledge and skill (Mainali, 2021). In other words, drawing quadratic graphs requires students to not only understand the underlying equations but also to translate that understanding into graphical representations.

Table 2 : Results for descriptive statistics for the test

Area of Concern	Items Number	Total Items	Total Marks	Mean	Standard deviation
Quadratic equation	1, 10, 11, 12, 13, and 14	6	10	5.72 (57.20%)	3.624
Quadratic inequalities	2, 3, 4 and 5	4	11	6.63 (60.27%)	2.575
Quadratic Graph	6, 7, 8, 9, 15 and 16	6	13	6.02 (46.31%)	3.760
Overall				**18.36**	**8.544**

Research Question 2: What are the students' perceptions towards their learning in the quadratic lessons?

The study's results were derived from a questionnaire focusing on key aspects of classroom involvement and attitudes towards learning. In terms of classroom involvement, it became evident that many students exhibited lower levels of engagement in the learning process. This was reflected in their responses, which indicated a preference for passive learning methods and a reluctance to participate actively. Specifically, students gave low ratings for their engagement in discovery-based learning activities (mean = 2.72) and active participation in answering questions (mean = 2.78). Furthermore, a significant number of students indicated a preference for remaining silent during classroom discussions (mean = 3.84).

In terms of attitudes towards learning, participants in the study expressed their perceptions through responses to the questionnaire. The data collected indicated that many participants found learning quadratic topics to be challenging. Some of the feedback included statements like "I found quadratic inequalities difficult, so I did not allocate much time to studying this section" (mean = 3.77) and "I found it difficult to interpret graphs of quadratic equations, which led me to not emphasise this topic as much" (mean = 3.62).

Ideally, the study highlights the importance of establishing clear connections among quadratic equations, inequalities, and graphs. The discovery method of learning is recognised as a highly effective approach for fostering these connections. It encourages active engagement among students, either individually or in study groups. However, despite the evident advantages of this approach, a considerable number of students tended to remain passive and disengaged, as evidenced by their consistently high ratings for silent participation and a reluctance to answer questions.

The findings emphasise the need for promoting active engagement among learners in discovery-based learning environments. In the context of mathematics education, the discovery method is widely acknowledged as the most effective means of conveying and reinforcing mathematical concepts (Kamaluddin & Widjajanti, 2019). Active learning within discovery-based environments encourages students to generate conjectures, develop hypotheses, and identify mathematical truths through deductive and inductive processes, as well as through careful observations (Prasad, 1970). Therefore, the study underscores the importance of incorporating active teaching methods to promote deeper understanding and connections among topics such as equations, inequalities, and graphs.

Research Question 3: What are students' misconceptions in answering the quadratic questions?
The findings aim to interpret students' abilities to answer the items in the three topics. Alternatively, their possible misconceptions were analysed (Table 3).

In the topic of **quadratic equations,** students focused on performing factorization. Their less acquisition was shown when they interpreted the concept of "satisfying the equations" (misconception 1 for quadratic equation). When the students were asked during the interview, "*What do you understand from the word roots or satisfies in this topic*?", most of them (example from R31) said, "*Not sure about that*". Furthermore, the respondents needed more clarity in their comprehension of the roots and solutions.

On the other hand, the awareness of working on quadratic equations towards the solutions involve the presentation of quadratic equations in standard or general forms that were not in their recalls. Misconception shown by R48 illustrates the confusion of representation in the form of fractions and the failure of representing an equation into a standard quadratic equation. R48 further expressed it as follows on his confusion about the denominator and numerator.

Researcher *: Okay, then why did you separate the numerator and denominator to create two new equations that are equal to x?*

R48 *: I just try to find the values of "x", but I don't know how. So, I just simply automatically did what was on my mind at that time.*

The above obstacles on students comprehend the meaning of factorization and when to conduct the factorization as well as simplifying a quadratic equation, which is written in fractional forms where the denominator is always more than zero. Strategies of highlighting were exemplified so that students are reminded to eliminate the denominator for writing it into a standard quadratic equation (Star et al., 2019). The obstacles are common among students who need to clarify the connection of solving equations via factorization with getting solutions or finding intersection points (solutions) of the function at $y = 0$. Hence, the algebraic properties of functions which highlight the difference between quadratic functions and equations need to be discovered (Aziz et al., 2018).

In terms of quadratic inequalities, some of the students struggled to convert an algebraic statement form into a graphical or diagram representation to determine the limit of the solution, such as x falls within a limited range, and others were unable to convert a graphical form into the corresponding algebraic form accurately. Specifically, R2's understanding of determining solutions for products of two expressions, namely $(2x + 1)$ and $(x + 2)$ for the inequality $(2x + 1)(x + 2) < 0$, displayed a failure to confirm the correct approach. To achieve a product with a negative value, the accepted values of x were found to be limited to the range $-2 < x < -0.5$, as depicted in Figure 1. However, R2 did not effectively convey the limits on the values of x for the multiplication results of $(2x + 1)$ and $(x + 2)$. Two common misunderstandings caused this mistake: firstly, R2 tended to think that $(2x + 1) < 0$ means something negative and $(x + 2) > 0$ means something positive, without thinking about how the two expressions affect each other. Secondly, R2 did not know how to use "and" and "or" correctly in the solution process.

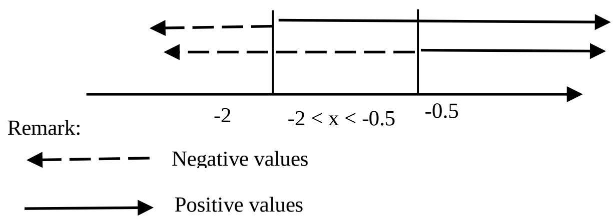


Figure 1 : Limit of the multiplication of two expressions

Additionally, this also seemed to have happened as a result of ambiguity regarding the meanings of "and" in terms of intersection, and "or" in terms of getting exclusive results. On the other hand, taking seriously managing ">" and "<" for the operations of expression in the inequalities also shows less acquisition of the understanding.

The students might fail to understand quadratic inequalities and how they vary from quadratic equations. Some students ignored the inequality indications and treated them as equations (Makonye & Shingirayi, 2014). It is critical to emphasize that quadratic inequalities entail the use of inequality signs (greater than, less than, greater than or equal to, less than or equal to) rather than equalities. A clear justification with reasoning for the allocation of values of solution is essential so that students differentiate and provide clear reason based on instrumental and relational knowledge is essential (Skemp, 1971). Teachers can effectively address this challenge by providing clear explanations of the subject matter and highlighting the fundamental distinctions between quadratic inequalities and equations. In this study, it became evident that the product of expressions like $(2x + 1)$ and $(x + 2)$ should be understood as the result of multiplying two numbers, and it becomes negative, represented as "< 0," after the multiplication process. Furthermore, as demonstrated in Misconception 2 regarding quadratic inequalities, it is crucial to draw comparisons involving the "or" operation.

In terms of **quadratic graphs**, there were a few obstacles that the students faced in solving the three topics. Only 23.5% and 38.2% of sample students responded correctly to item 6 and item 7, respectively. These two items were designed to test students' conceptual understanding of the root of the quadratic as well as the parabolic intersection. The purpose of item 7 is to assess students' ability to relate the number of locations at which the graph of the equation $y = ax^2 + bx + c$ intersects the axes with the value of the real roots of the equation $ax^2 + bx + c = 0$. First and foremost, the idea that a parabola must cut the *y*-axis, even though it might or might not cut the x-axis, was not understood by some students. Table 3 (misconception 1 for quadratic graph 1) shows the student's (R77) responses related to **misconceptions in understanding of "roots".** The findings of this study revealed that students meant "root" as "intersection on the *y*-axis and *x*-axis. This student believed that every parabolic graph that intersects the y-axis also **intersects x-axis and has two roots.** Other than that, some students believe that $y = ax^2 + bx + c$ should have two unique real roots in order to intersect with the axis (x-axis or y-axis) at two locations. Some of them even agreed that if the equation $ax^2 + bx + c = 0$ has two distinct real roots, the graph should intersect **either the y-axis or x-axis** two times, as described in her excerpt below.

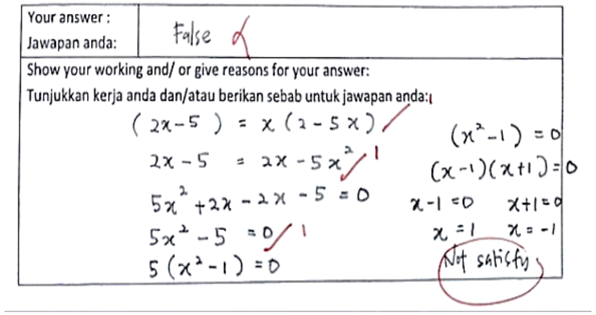
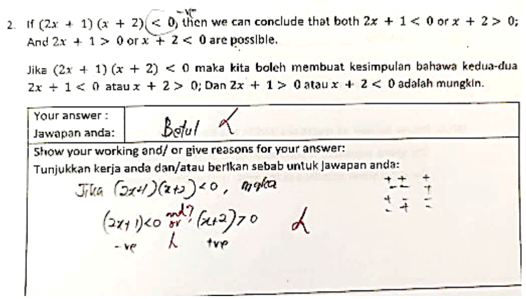
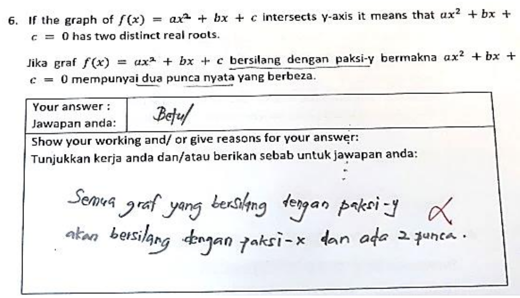
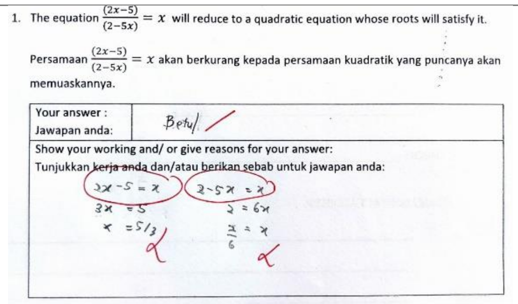
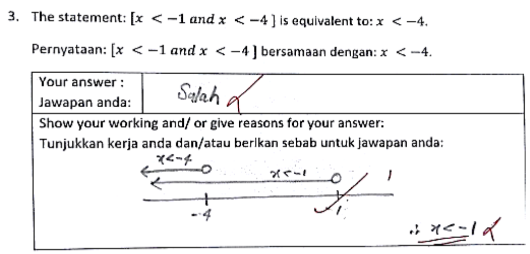
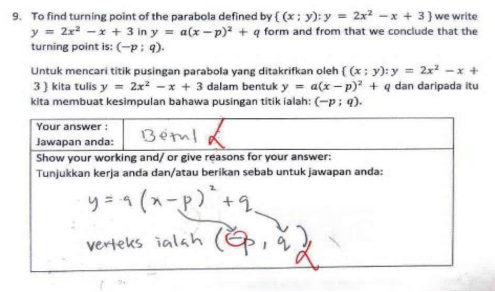
Researcher	*: What do you understand from the statement if a graph of $y = ax^2 + bx + c$ intersects y-axis it means that the equation has two distinct real roots?*
R77	*: I understand that that quadratic graph will intersect y-axis at two points if it has two distinct real roots.*
Researcher	*: How about if it intersects x-axis at two points?*
R77	*: The same thing happens. As long as it intersects two points at x-axis or y-axis, thus there will be two distinct real roots that satisfy the equation.*

R77 presented an explanation for the graph's creation; however, the provided justification is not acceptable. This feedback serves as a valuable opportunity for students to enhance their understanding with expert guidance. Encouraging further discussion and reasoning in the learning process is strongly encouraged (Vedran et al., 2019).

On the other hand, completing the square and vertex conceptualization has been seen as a procedure in graphing rather than focusing on the interpretation of getting a turning point (Kotsopoulos, 2007)**.** The finding in Table 3 (misconception 1 for quadratic graph 2) shows that the student has wrongly employed the negative sign from the vertex form of the equation to conclude the vertex or the turning point of the parabola. When asked during an interview, the student (R94) response indicate the vertex as $(-p, q)$ since $y = a(x - p)^2 + q$ without understanding that the minimum point when $y = p$ is resulted from $(x - p) = 0$, where to get the minimum point $(x - p)^2$ must be the minimum, namely $(x - p)^2$ is impossible to be negative, hence $(x - p)^2 = 0$ as R94 is described as below:

"I'm aware that standard form of quadratic equation can be transformed into vertex form of quadratic equation through completing the square. But I forgot how to get the turning point from the vertex form of quadratic equation, and since the statement show that the turning point was $(-p, q)$, so I just simply agree to it. And I got a little bit confused because from what I memorised; the vertex form of quadratic equations was $y = a(x-h)^2 + k$".

Table 3 : Misconceptions in learning quadratic topics.

Misconception	Quadratic Equations	Quadratic Inequalities	Quadratic Graphs
1	 Your answer : Jawapan anda: False Show your working and/ or give reasons for your answer: Tunjukkan kerja anda dan/atau berikan sebab untuk jawapan anda: $(2x-5) = x(2-5x)$; $2x-5 = 2x-5x^2$; $5x^2+2x-2x-5=0$; $5x^2-5=0$; $5(x^2-1)=0$; $(x^2-1)=0$; $(x-1)(x+1)=0$; $x-1=0$, $x+1=0$; $x=1$, $x=-1$; Not satisfy R31	 2. If $(2x+1)(x+2) < 0$, then we can conclude that both $2x+1<0$ or $x+2>0$; And $2x+1>0$ or $x+2<0$ are possible. Jika $(2x+1)(x+2)<0$ maka kita boleh membuat kesimpulan bahawa kedua-dua $2x+1<0$ atau $x+2>0$; Dan $2x+1>0$ atau $x+2<0$ adalah mungkin. Your answer : Jawapan anda: Betul Show your working and/ or give reasons for your answer: Tunjukkan kerja anda dan/atau berikan sebab untuk jawapan anda: Jika $(2x+1)(x+2)<0$, maka $(2x+1)<0$ and/or $(x+2)>0$ R2	 6. If the graph of $f(x) = ax^2+bx+c$ intersects y-axis it means that $ax^2+bx+c=0$ has two distinct real roots. Jika graf $f(x) = ax^2+bx+c$ bersilang dengan paksi-y bermakna $ax^2+bx+c=0$ mempunyai dua punca nyata yang berbeza. Your answer : Jawapan anda: Betul Show your working and/ or give reasons for your answer: Tunjukkan kerja anda dan/atau berikan sebab untuk jawapan anda: Semua graf yang bersilang dengan paksi-y akan bersilang dengan paksi-x dan ada 2 punca. R77
2	 1. The equation $\frac{(2x-5)}{(2-5x)} = x$ will reduce to a quadratic equation whose roots will satisfy it. Persamaan $\frac{(2x-5)}{(2-5x)} = x$ akan berkurang kepada persamaan kuadratik yang puncanya akan memuaskannya. Your answer : Jawapan anda: Betul Show your working and/ or give reasons for your answer: Tunjukkan kerja anda dan/atau berikan sebab untuk jawapan anda: $2x-5=x$; $3x=5$; $x=5/3$ $2-5x=x$; $2=6x$; $\frac{2}{6}=x$ R48	 3. The statement: $[x<-1$ and $x<-4]$ is equivalent to: $x<-4$. Pernyataan: $[x<-1$ and $x<-4]$ bersamaan dengan: $x<-4$. Your answer : Jawapan anda: Salah Show your working and/ or give reasons for your answer: Tunjukkan kerja anda dan/atau berikan sebab untuk jawapan anda: $x<-4$, $x<-1$, -4; $\therefore x<-1$ R2	 9. To find turning point of the parabola defined by $\{(x;y): y = 2x^2-x+3\}$ we write $y = 2x^2-x+3$ in $y = a(x-p)^2+q$ form and from that we conclude that the turning point is: $(-p; q)$. Untuk mencari titik pusingan parabola yang ditakrifkan oleh $\{(x;y): y = 2x^2-x+3\}$ kita tulis $y = 2x^2-x+3$ dalam bentuk $y = a(x-p)^2+q$ dan daripada itu kita membuat kesimpulan bahawa pusingan titik ialah: $(-p; q)$. Your answer : Jawapan anda: Betul Show your working and/ or give reasons for your answer: Tunjukkan kerja anda dan/atau berikan sebab untuk jawapan anda: $y = a(x-p)^2+q$; verteks ialah $(-p, q)$ R94

Remark: "R" indicates "respondent"; "R31" is the 31st respondent

The findings revealed that the students rely solely on memorizing formulas without understanding the underlying concepts. Memorizing formulas can be helpful to some extent, but true comprehension and problem-solving skills come from understanding the principles and relationships behind those formulas. By focusing on comprehension and critical thinking, students will become more adaptable and confident when dealing with changes in coefficients or applying formulas in various contexts. Other than that, the student also failed to recall whether the coefficient p, which indicates x-coordinate, should be negative or not for $a > 0$.

CONCLUSION

In conclusion, when it comes to teaching quadratic topics, teachers must prioritise the establishment of strong connections among the three core components: quadratic equations, quadratic inequalities, and quadratic functions. These three aspects are intricately intertwined, and fostering a holistic understanding of quadratic mathematics necessitates a cohesive approach. Besides, the active encouragement of classroom discussions is a crucial tactic that teachers should use. These discussions should revolve around essential aspects of quadratic mathematics, with a particular emphasis on the graphical representation of these concepts. It is through such discourse that students can bridge the gap between their existing algebraic knowledge and their graphical understanding of quadratic functions. Furthermore, teachers should take a proactive role in guiding students' knowledge acquisition. This involves assisting students in not only determining solutions to quadratic functions but also in establishing a meaningful connection between these algebraic solutions and their graphical counterparts. Emphasising the concept of intersection points, which algebraic solutions represent in graphical representations, is particularly valuable in deepening students' comprehension of quadratic topics.

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