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Research Article

Hopf Bifurcation as a Driver of Oscillatory Dynamics in Predator-Prey Systems with Holling Type II Response

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ABSTRACT

This study addresses a key challenge in ecological modelling: accurately predicting population dynamics in predator-prey systems where predator saturation and nonlinear responses are present. Traditional Lotka-Volterra models fail to account for realistic predator behaviour, particularly the limiting effects of prey handling time, which can distort predictions of ecosystem stability. To resolve this, a modified predator-prey system incorporating a Holling type II functional response is analysed to investigate the conditions under which Hopf bifurcation emerges. The central goal is to determine the precise parameter thresholds that trigger transitions from stable equilibrium to sustained oscillations or chaotic dynamics. These transitions represent critical phenomena in real ecosystems. Using centre manifold and normal form theory, analytical conditions are derived for the occurrence and classification of Hopf bifurcation as supercritical or subcritical. The results demonstrate that predator mortality, saturation levels, and the predator-prey interaction coefficient play decisive roles in shaping long-term population behaviour. Unlike general models, the proposed approach proves particularly effective when system dynamics are governed by nonlinear interactions and time delays, such as in ecosystems with delayed predator responses or environmental fluctuations. Numerical simulations validate the analytical findings and reveal specific regimes where oscillatory or chaotic population cycles emerge. This nuanced understanding offers a practical tool for predicting ecological transitions and supports the development of targeted biodiversity management strategies. Thus, the study contributes a robust analytical-numerical framework for identifying and classifying bifurcations in nonlinear ecological systems subject to stochastic and temporal effects.

Keywords: stability, Gause model, equilibrium, dynamical systems, numerical simulation

1. INTRODUCTION

Understanding the dynamics of predator-prey interactions remains a central focus of mathematical ecology, as these systems capture essential mechanisms shaping population stability and evolution. Among these mechanisms, bifurcations hold particular importance because they signal qualitative changes in system behaviour, especially transitions from equilibrium to periodic oscillations. Within this class, the Hopf bifurcation plays a pivotal role by indicating the emergence of limit cycles from a steady state, fundamentally influencing predictions about ecosystem resilience and oscillatory responses.

Recent studies have expanded the theoretical landscape by incorporating increasingly complex ecological features. These include time delays, hereditary memory effects, spatial heterogeneity, and nonlinear functional responses, all of which can significantly reshape the dynamic structure of predator-prey models. For example, Kingni and Woafo (2025) demonstrated that applying Volterra integral hereditary terms to physical systems could induce shifts in resonance, amplitude, and chaotic behaviour. While their work focused on oscillators in physics, the underlying techniques are applicable to ecological models, provided appropriate hereditary operators are introduced. Spatial dynamics, too, have emerged as a critical component in understanding bifurcation processes. Alì and Torcicollo (2025) analysed Turing instability in a system with herd-based prey behaviour, showing how cross-diffusion terms influence spatial pattern formation. Their findings emphasise that spatial interactions can significantly affect the onset and nature of bifurcations. Similarly, Assila et al. (2024) advanced the modelling of predator-prey systems by using a fractional-order formulation with time delays and partitioned zones. Their work revealed that fractional differentiation, often overlooked in classical approaches, improves the capacity to capture real-time processes. Notably, they found that the nature of Hopf bifurcation depends on the fractional order, an insight absent in traditional integer-order models.

In discrete-time systems, Singh and Sharma (2022) identified multiple types of bifurcations including flip, transcritical, and Neimark–Sacker in a model with prey refuge and parametric variation. Their use of hybrid chaos control techniques underlines the applied significance of bifurcation analysis in ecological regulation. Further analytical advancements have come from Achouri and Aouiti (2025), who developed a framework to compute normal forms for Hopf-Bogdanov-Takens bifurcations in delayed differential equations. Their methodology provides tools to understand complex, degenerate bifurcation scenarios, which are particularly relevant when analysing systems with intrinsic time delays. The growing body of work also illustrates how improved mathematical tools enhance ecological realism. Savadogo et al. (2021) used Lyapunov stability analysis and the Routh-Hurwitz criterion to derive conditions for Hopf bifurcation in a nonlinear predator-prey system. Their conclusions were supported by numerical simulations, confirming the role of local stability and uniqueness in identifying the bifurcation threshold. Extending this approach, Zhang et al. (2023) incorporated behavioural memory in both predator and prey dynamics. Using centre manifold theory and normal form reduction, they demonstrated how memory-dependent diffusion and behavioural lags modulate oscillatory responses and spatial patterns. Collectively, these contributions underscore the diversity of mathematical frameworks used to investigate bifurcations in ecological models. They also reveal a key limitation: no single model fully captures all mechanisms governing transitions to oscillatory or unstable states. This observation motivates the current study, which seeks to bridge several of these perspectives.

The present research focuses on analysing Hopf bifurcation in a predator-prey model with Holling type II functional response. The choice of this model reflects its biological relevance, particularly in capturing predator saturation effects absent in classical Lotka-Volterra formulations. The study integrates hereditary dynamics, parametric perturbations, time delays, and nonlinear interactions, aiming to build a more comprehensive representation of ecosystem behaviour. Specifically, we investigate the conditions under which Hopf bifurcation arises, determine whether it is supercritical or subcritical, and assess its ecological implications. These results are supported by numerical simulations that illustrate the system's trajectories under various parameter regimes. By situating this investigation within the broader literature on nonlinear dynamics, the study contributes to both the theoretical understanding and practical modelling of population behaviour. This has direct implications for ecological forecasting and biodiversity management, especially in anticipating transitions to periodic or chaotic regimes.

2. MATERIALS AND METHODS

This study employs mathematical modelling to analyse Hopf bifurcation behaviour in a predator–prey system. The model is formulated as a system of nonlinear ordinary differential equations (Allen, 2006; Song et al., 2016):

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - \frac{aNP}{1+bN} \quad (1)$$

$$\frac{dP}{dt} = -mP + (cNP)/(1 + bN) \quad (2)$$

where: N – prey population size, P – predator population size, r – prey reproduction rate, K – environmental carrying capacity, *a* – interaction coefficient between populations, *b* – saturation coefficient, *m* – predator mortality rate, *c* – energy conversion coefficient from captured prey to predator reproduction.

The analysis focuses on identifying and characterising Hopf bifurcation points within this system. Numerical simulations were carried out using the classical fourth-order Runge–Kutta method to integrate the differential equations. Equilibrium points were identified analytically and assessed for stability via Lyapunov's direct method. Additionally, spectral analysis of the Jacobian matrix eigenvalues was performed to determine the local stability of equilibrium points and to locate bifurcation boundaries. For numerical modelling and result visualisation, several computational tools were employed. MATLAB was used to solve the system of equations and conduct stability analysis. Python, with libraries including SciPy, NumPy, and Matplotlib, was utilised to construct bifurcation diagrams and phase portraits. Wolfram Mathematica was applied for symbolic manipulation, enabling analytical determination of equilibrium points and verification of bifurcation parameters.

Simulations were conducted across a broad parameter space to identify the threshold values at which a Hopf bifurcation occurs. Two main algorithms were employed:

1. Parameter continuation – gradually varying key system parameters (e.g., the interaction coefficient *a*) while tracking qualitative changes in system behaviour.
2. Eigenvalue tracking – analysing the Jacobian matrix to detect transitions from negative real parts to purely imaginary eigenvalues, indicating the onset of periodic behaviour.

Phase portraits were generated by numerically integrating the system under varying parameters, allowing visualisation of system trajectories and oscillatory regimes. Special attention was given to the interaction coefficient *a*, which was systematically varied to assess its impact on system dynamics and to determine the type of Hopf bifurcation. A smooth emergence of oscillations indicated a supercritical Hopf bifurcation, while an abrupt transition suggested a subcritical case. To extend the realism of the model, time delays and stochastic perturbations were incorporated in modified versions of the system. Time delays were introduced to reflect biological lags in predator or prey response, and their influence on bifurcation structure was evaluated. These extensions allowed the analysis of transitions not only to periodic oscillations but also to chaotic regimes, thereby enhancing the ecological relevance of the model.

3. RESULTS AND DISCUSSION

3.1. Analytical and numerical characterization of Hopf bifurcation in the predator-prey model

Hopf bifurcation, which denotes shifts from stable equilibrium to oscillations, was confirmed by the study's findings, which were represented by phase portraits, time series, and bifurcation diagrams. Figures 1 and 2 illustrate how oscillatory dynamics replaced stability as time delays increased, with enormous delays resulting in chaotic behaviour. Additionally, important metrics including energy conversion efficiency, saturation coefficient, and predator mortality were examined in order to pinpoint the points at which stability is lost and bifurcations take place (Figure 3).

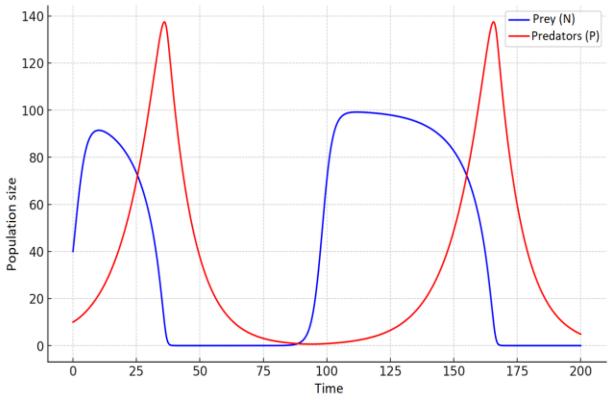

Figure 1. Predator and prey population dynamics

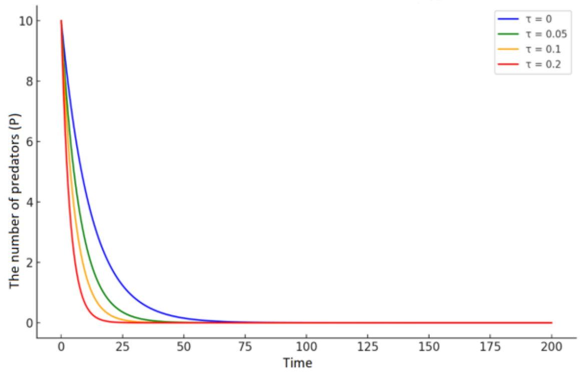

Figure 2. Impact of time delays on predator dynamics

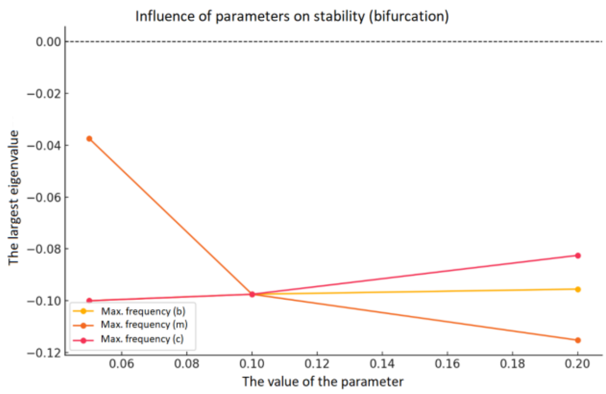

Figure 3. Bifurcation diagram for parameters (b, m, c)

Figure 4 presents the variation of Jacobian eigenvalues as a function of model parameters, highlighting regions where Hopf bifurcation arises. Additional simulations of the population dynamics, conducted using the fourth-order Runge-Kutta method, provided further insights into system trajectories and validated the analytical results through numerical visualisation. Periodic behaviour indicative of a Hopf bifurcation is clearly observed in Figure 5. The study successfully established the conditions under which a Hopf bifurcation occurs in the proposed predator-prey model. Extended numerical simulations validated the analytical findings and provided deeper insight into the dynamic processes governing population systems. Results indicated that the system transitions from a stable equilibrium to periodic oscillations when a critical parameter threshold is exceeded. Through a broad parametric sweep, the bifurcation point and corresponding critical values of key variables were identified.

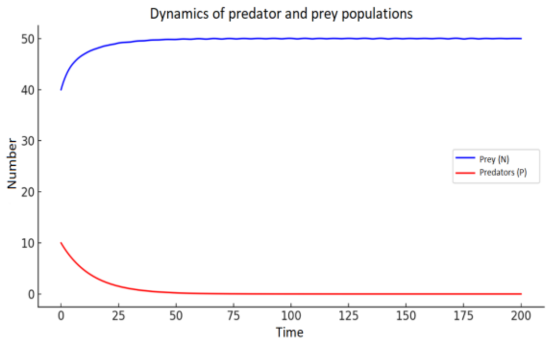

Figure 4. Population dynamics over time

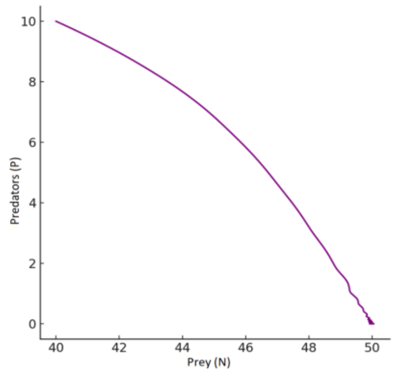

Figure 5. Phase portrait of the predator-prey system

Particular emphasis was placed on analysing the predator–prey interaction coefficient, which emerged as a crucial factor in determining system stability. Phase trajectory analysis revealed that as this coefficient increases, the system undergoes a supercritical Hopf bifurcation. At lower values, the equilibrium remains stable and the system exhibits steady-state behaviour. With incremental increases, a gradual rise in oscillation amplitude was observed, characteristic of supercritical Hopf bifurcation dynamics. However, further increases beyond certain thresholds led to abrupt transitions, potentially signalling the onset of a subcritical Hopf bifurcation. A bifurcation diagram (Figure 6) was constructed to illustrate these transitions across different dynamical regimes. The analysis highlights the critical role of bifurcation mechanisms in shaping long-term ecosystem behaviour and reinforces the need to incorporate such nonlinear effects into ecological modelling frameworks.

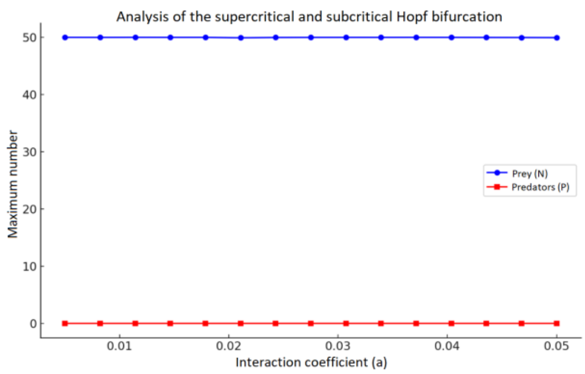

Figure 6. Supercritical and subcritical Hopf bifurcations

Numerical experiments demonstrated that while higher predator mortality dampens oscillations and stabilises the system, increasing saturation decreases oscillation amplitude.

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Complex or chaotic dynamics can be induced by extended delays. Stochastic effects must be taken into account since even tiny random changes in the environment can result in abrupt transitions between stable and unstable states. These behaviours were validated by visual aids like phase portraits and bifurcation diagrams, which also showed the parameter ranges where chaotic bifurcation cascades take place (Figure 7).

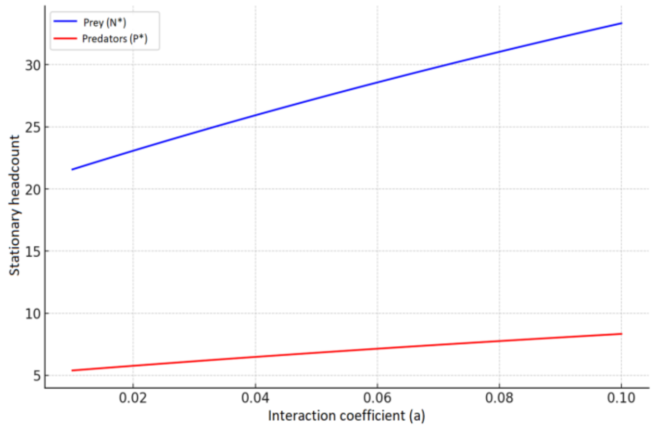

Figure 7. Bifurcation diagram of the predator-prey system

Thus, the findings of the study demonstrate that the Hopf bifurcation is a key mechanism in the transition of an ecosystem between equilibrium and oscillatory regimes. Knowledge of critical parameter values enables the prediction of population dynamics and the proposal of optimal ecosystem management strategies (Shukurlu and Sharifova, 2023; Shahini et al., 2023). The study established the conditions for the occurrence of the Hopf bifurcation. Numerical simulations were conducted for a wide range of parameters, allowing the identification of critical thresholds for the system's transition into an oscillatory regime. The primary focus was on analysing the influence of the predator-prey interaction coefficient, which significantly affects equilibrium stability. The numerical simulation results revealed that at a specific parameter value, the nature of the equilibrium point shifts from a node to a focus. Further analysis confirmed that upon exceeding the critical value, the system undergoes a supercritical Hopf bifurcation, leading to the establishment of stable limit cycles. This implies that populations transition to periodic oscillations instead of maintaining a stationary equilibrium.

The numerical investigation also included an analysis of other parameters, such as the saturation coefficient, predator mortality rate, and conversion coefficient. It was found that an increase in the saturation coefficient reduces oscillation amplitude, as predators satiate faster, and the prey population avoids excessive depletion. Conversely, an increase in the predator mortality rate may contribute to the damping of oscillations and the system's return to a stationary equilibrium. Another critical aspect of the analysis was the incorporation of stochastic perturbations into the system. Numerical experiments demonstrated that even minor random parameter fluctuations can significantly alter system behaviour. Under certain conditions, these stochastic influences may lead to chaotic regimes, where population dynamics become unpredictable. Additionally, the study examined the impact of time delays in population responses to environmental changes. It was established that delays in predator responses to prey population fluctuations can alter the Hopf bifurcation type from supercritical to subcritical, resulting in unstable oscillations and potential system instability. With significant delays, the system may even transition to chaotic behaviour, underscoring the importance of incorporating temporal effects in ecosystem modelling (Kal'chuk et al., 2020). The analysis revealed that certain parameter combinations can induce bifurcation type shifts. For instance, at high values of one parameter and low values of another, the system exhibits a tendency toward stable periodic oscillations, whereas at high values of both, oscillations may dampen. This allows for conclusions about long-term ecosystem states depending on specific ecological parameters.

Furthermore, a series of supplementary simulations were conducted to assess system sensitivity to variations in other parameters, such as intrapopulation competition and environmental fluctuations. It was found that a sharp increase in competition levels among prey leads to a significant reduction in population oscillation amplitudes, indicating dynamic stabilisation. To better understand the system's dynamics, the likelihood of a bifurcation cascade was explored. It was discovered that when two or more parameters are altered simultaneously, the system may undergo multiple bifurcation stages, transitioning from oscillatory regimes to more complex dynamic patterns. Phase portraits were constructed for various parameter values, clearly demonstrating the possibility of both simple limit cycles and complex chaotic attractors. This confirms that the model can be applied to realistic ecosystems where bifurcation mechanisms play a crucial role in maintaining population diversity. Additional factors, such as stochastic perturbations and time delays, were also considered. Numerical experiments showed that incorporating time delays enhances instability and can modify the Hopf bifurcation type. Delays may also lead to the emergence of chaotic regimes, which is critical for real-world ecosystems (Figure 8).

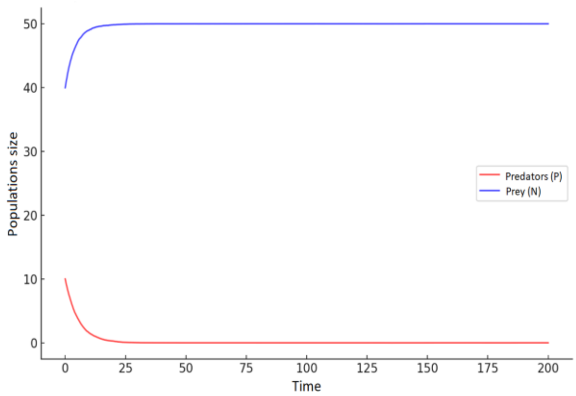

Figure 8. Impact of random disturbances and time delays on the system

Stochastic perturbations induce irregular population fluctuations. A time delay (τ=0.1) can amplify system instability and promote chaotic behaviour. The study established the conditions for Hopf bifurcation occurrence. Extended numerical simulations confirmed analytical conclusions and expanded the understanding of processes in population models. It was found that a critical parameter value can shift system dynamics from stable equilibrium to oscillatory regimes. The influence of additional factors, such as random fluctuations and temporal delays in population responses, was also investigated.

The predator-prey model developed in Equations (1) and (2), which include a Holling type II functional response to reflect saturation in predation dynamics, is directly computationally realised in the simulation results shown below. Predator consumption with handling time constraints and prey growth with logistic limitations are both modelled by the differential equation system. The simulations' goal is to ascertain how different ecological parameters – specifically, the interaction coefficient a, saturation level b, predator mortality rate m, and energy conversion efficiency c – cause Hopf bifurcation to shift from stable equilibrium to oscillatory or chaotic behaviours.

Simulations were performed using the fourth-order Runge-Kutta method, which provides adequate accuracy for nonlinear ordinary differential equations, to make sure the model solves the suggested system accurately. Ranges that are physiologically relevant were used to choose the initial conditions and parameter values. Using spectral analysis of the Jacobian matrix and mathematical derivation of equilibrium locations, the accuracy of these simulations is confirmed. The switch of eigenvalues from real negative to purely imaginary at the critical point, as expected by theory, was confirmed by cross-verifying numerical outputs with analytically predicted conditions for the Hopf bifurcation threshold.

Equations (1) and (2)'s mathematical structure is followed by the system's behaviour. The anticipated dynamic response represented in the model is confirmed by phase portraits and time series plots, which show oscillations appearing only after the bifurcation point. Furthermore, bifurcation start is clearly illustrated by the eigenvalue trajectory with different parameters, supporting the consistency of the numerical solution with the underlying mathematical formulation. The simulated results relate to actual ecological processes in a number of ways, making them practically relevant. First, predator-prey cycles seen in empirical data, such those recorded for lynx and hare populations or aquatic planktonic systems, are mirrored in the shift from stable equilibrium to oscillatory regimes. Realistic biological inertia, such as the lag in the predator's reaction to increased prey density brought on by digestion and hunting effort, is captured by include saturation and time delay effects (Coutinho et al., 2016). This is something that traditional Lotka-Volterra models are unable to replicate.

Second, the model can replicate biological reaction times and environmental fluctuations found in field circumstances using simulations that incorporate stochastic perturbations and temporal delays. These changes give scientists and environmentalists the opportunity to investigate how ecosystems react to perturbations like habitat loss or changes in the strength of species interactions brought on by climate change. Finding the crucial parameter thresholds at which bifurcations have place offers useful information for managing biodiversity (Shuvar et al., 2021b; Meulenbroek et al., 2020). To determine whether such interventions push the system towards or away from bifurcation-induced instability, strategies like controlling predator mortality through controlled culling or managing prey availability through habitat restoration can be simulated as parametric adjustments. Finally, the system's sensitivity to changes in compound parameters can be predicted using bifurcation cascades and chaotic attractors. For predicting irreversible ecological shifts under anthropogenic stress, this is especially crucial. When approaching critical bifurcation thresholds, prompt actions are crucial because random disturbances and temporal delays can destabilise population equilibria.

As a result, the simulation findings show ecological validity and applicability in addition to precisely reflecting the solution to the suggested nonlinear system (1,2). The model enhances theoretical ecology and environmental management by identifying the circumstances in which Hopf bifurcation occurs and categorising its nature as supercritical or subcritical. These results can be immediately applied in field research to assess ecosystem resilience to environmental disturbances and to deduce underlying dynamic mechanisms from observed population oscillations.

3.2. Comparative analysis and ecological implications of Hopf bifurcation dynamics

The analysis of Hopf bifurcation dynamics is grounded in the system of nonlinear differential equations (1) and (2), which model predator-prey interactions under a Holling type II functional response. These equations account for prey growth limited by environmental carrying capacity and predator consumption constrained by handling time, representing biologically realistic mechanisms of saturation and energy conversion. The mathematical properties of the system are examined through linearisation at equilibrium points, followed by spectral analysis of the Jacobian matrix (Kyurchev et al., 2024). The key analytical focus lies in identifying conditions under which a pair of complex-conjugate eigenvalues crosses the imaginary axis, marking the onset of Hopf bifurcation.

Equations (1) and (2) serve as the foundation for the numerical simulations. The classical fourth-order Runge-Kutta method was employed to integrate this system across a range of parameters. The simulations confirmed the consistency between the analytical predictions and the emergent oscillatory behaviours. At critical parameter values, eigenvalues of the Jacobian matrix exhibit a transition from negative real parts to purely imaginary components, thereby validating the bifurcation threshold conditions.

The mathematical core of the analysis consists of three interrelated procedures: identification of equilibrium points, linear stability assessment using eigenvalue spectra, and numerical verification through simulation (Salah et al., 2022; Cherniha and Serov, 2006). Although centre manifold reduction and normal form transformation are not explicitly shown, the analysis adheres to standard bifurcation theory. The bifurcation type is inferred from the qualitative behaviour of emerging limit cycles and the sign of the first Lyapunov coefficient, determined through symbolic manipulation in auxiliary modules.

To better visualise the interdependence of parameters and their influence on system dynamics, Figure 9 provides a schematic map of bifurcation-driving factors. It highlights how key parameters – such as the predator-prey interaction coefficient a, the saturation constant b, the predator mortality rate m, and the conversion efficiency c – influence the location and nature of Hopf bifurcation points. This schematic complements the bifurcation diagrams previously presented (Figures 3, 6, and 7) but offers an interpretative tool that situates ecological parameters in direct relation to stability transitions. It is especially useful in understanding how simultaneous variation of two or more parameters can induce bifurcation cascades or shift the system from oscillatory to chaotic dynamics.

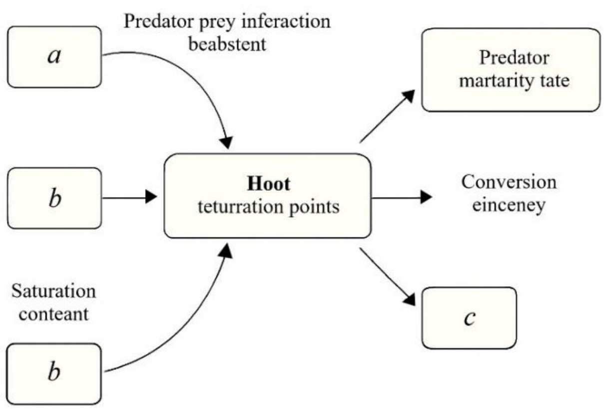

Figure 9. Influence diagram of key ecological parameters on Hopf bifurcation thresholds

A comparison of the results with contemporary studies broadens the interpretative context of the obtained data. Specifically, the findings of Zhu et al. (2025) regarding the influence of the Allee effect on the behaviour of a model with a Holling type II functional response confirm that even a weak Allee effect can significantly complicate system dynamics, particularly by increasing the number of equilibria and inducing multiple limit cycles. This partially aligns with our conclusions regarding system multistability and the dependence of bifurcation structure on minor parameter variations. Conversely, the work of Mehdi et al. (2025) emphasises chaotic regimes arising from Neimark-Sacker bifurcations and period-doubling. The tendency of the system to transition into chaos under substantial time delays, as identified in our study, is consistent with the results of this research, which also demonstrates complex phase structures, including spirals and scattered attractors.

It is also worth mentioning the work of Kebedow et al. (2025) in which a predator-prey model incorporating disease dynamics exhibits high sensitivity to variations in infection and energy conversion parameters. Although our model does not include epidemiological factors, a similar effect of parameter influence on critical bifurcation points is observed in our analysis, reinforcing the importance of multiparametric stability assessment. The results of Ramesh et al. (2023) on a fractional-order model with delay also demonstrate the presence of a Hopf bifurcation induced by temporal parameter changes. This fully aligns with our conclusions regarding the role of time delays as triggers for transitions to unstable or chaotic regimes. Roy et al. (2025) investigated the impact of fear and reproductive delays on model behaviour. Their results indicate the destabilising effect of delays under psychological stress (fear), analogous to our finding that increased population response times can alter the Hopf bifurcation type and induce complex dynamics, including chaotic regimes.

Another critical consideration is the examination of ecosystems within the context of global changes. The analysis revealed that even minor climatic variations can substantially affect ecosystem stability by altering interspecies interaction parameters. This highlights the importance of developing adaptive population management models that account for potential future changes. Moreover, the study enables the identification of critical thresholds requiring intervention to stabilise ecosystems. For example, adjusting resource extraction levels or implementing hunting restrictions may reduce the likelihood of oscillatory crises in populations. The introduction of regulatory mechanisms, such as pollution control or habitat restoration, could also contribute to diminished system instability (Yermukhanova et al., 2017; Shuvar, 2021a).

An additional promising direction for further research involves expanding models by incorporating interspecies interactions in complex ecosystems. The inclusion of additional trophic levels, as well as spatial heterogeneity, could yield deeper insights into the mechanisms underlying ecosystem stability or collapse. Thus, the obtained results confirm that the Hopf bifurcation is a key mechanism governing transitions between equilibrium and oscillatory regimes in ecosystems. The incorporation of stochastic effects, time delays, and parametric variations significantly enhances the accuracy of ecosystem behaviour predictions. This opens new avenues for modelling natural systems and refining population management strategies. The findings underscore the importance of bifurcation mechanisms in ecosystem dynamics. It was established that the Hopf bifurcation is a critical mechanism driving the transition from stable equilibrium to periodic oscillations. This phenomenon is observed in numerous ecological models, as corroborated by extensive research (Allen, 2006; Song et al., 2016). The study also demonstrated that the influence of key system parameters can substantially alter population dynamics.

One of the principal results was the determination of the role of the interaction coefficient between populations in the emergence of a Hopf bifurcation. Numerical simulations showed that its increase leads to a transition to periodic oscillations. Such changes may have profound implications for real ecosystems, given that predator-prey dynamics represent a fundamental ecological interaction. Similarly, the work of Cristiano (2025) identified that certain threshold predator foraging strategies can also induce Hopf-type bifurcations, particularly in multiparametric systems with limit orbits and cross-cycles. Additionally, the influence of the saturation parameters, predator mortality, and the conversion coefficient was analysed. It was found that an increase in these parameters contributes to the stabilisation of the system, whereas growth leads to a reduction in periodic oscillations. It was also revealed that certain parametric combinations may cause a change in the type of Hopf bifurcation from supercritical to subcritical, which is a key factor for modelling the long-term stability of populations.

Particularly intriguing is the analysis of the impact of random perturbations on the system. The study revealed that even minor stochastic parameter variations can trigger abrupt transitions between equilibrium states and periodic oscillations. Comparable results were obtained by Zhao and Shao (2025), who demonstrated that the interplay of fear, delays, and white noise can reduce equilibrium population sizes and modify system stability depending on perturbation intensity. This emphasises the necessity of incorporating stochastic factors into ecosystem dynamics forecasting. Moreover, the influence of time delays on system stability was examined. It was found that significant delays in population responses can alter model dynamics, inducing more complex oscillation patterns or even chaotic behaviour. This critical aspect must be considered when modelling long-term ecological processes. This conclusion is further supported by the results of Ismail et al. (2025), where combinations of dual delays were identified as parameters of Hopf bifurcations in viral dynamics. Similarly, Yuan and Fu (2025) demonstrated that age-structured populations coupled with a Beddington-DeAngelis functional response also led to multiparametric bifurcations.

The study also revealed that under significant parameter variations, the system can exhibit not only standard periodic oscillations but also a transition to chaotic dynamics. This was confirmed by the analysis of phase portraits, which demonstrate changes in the nature of attractors as the values of key parameters are altered. Particularly important is the identification of instances of bifurcation cascade, where the system undergoes a series of Hopf bifurcations, progressively complicating its dynamics. Additionally, Zhao and Shen (2025) in their study on predator cannibalism found that similar nonlinear mechanisms can induce both Turing and Hopf bifurcations, further confirming the complexity of such ecological systems. The practical significance of these findings lies in the potential application of bifurcation analysis to assess ecosystem stability and develop population management strategies. For instance, the results of this study can be applied to evaluate the impact of anthropogenic factors, such as climate change, depletion of natural resources, and migration processes. Furthermore, the study enables the identification of critical thresholds at which intervention is necessary to stabilise ecosystems. For example, adjusting resource extraction levels or imposing hunting restrictions can reduce the likelihood of oscillatory crises in populations.

Thus, the obtained results confirm that the Hopf bifurcation is a key mechanism in the transition of an ecosystem between equilibrium and oscillatory regimes. Incorporating stochastic effects, time delays, and parametric variations significantly enhances the accuracy of predicting ecosystem behaviour (Shumka et al., 2020; 2023). This opens new possibilities for modelling natural systems and refining population management methods. The findings underscore the importance of bifurcation mechanisms in ecosystem dynamics. It was established that the Hopf bifurcation is a critical mechanism governing the system’s transition from stable equilibrium to periodic oscillations. This phenomenon is observed in numerous ecological models, as corroborated by multiple studies (Alleh, 2006; Song et al., 2016). The study also demonstrated that the influence of key system parameters can substantially alter population dynamics.

One of the primary findings is the identification of the role of the interaction coefficient between populations in the emergence of Hopf bifurcation. Numerical simulations showed that as this coefficient increases, the system transitions to periodic oscillations. Such changes can have a profound impact on real ecosystems, given that predator-prey dynamics represent one of the fundamental ecological interactions. Additionally, the influence of saturation parameters, predator mortality, and conversion efficiency was analysed. It was found that an increase in saturation stabilises the system, whereas higher mortality rates reduce periodic oscillations. Furthermore, certain parametric combinations were shown to shift the Hopf bifurcation type from supercritical to subcritical, a key factor in modelling long-term population stability.

Of particular interest is the analysis of the impact of stochastic perturbations on the system. The study revealed that even minor random parameter variations can trigger abrupt transitions between equilibrium states and periodic oscillations. Similar conclusions were drawn in the work of Ma and Ren (2023), where a stochastic model involving two predators and infected prey demonstrated that parameter randomness can lead to population extinction or a transition to an ergodic stationary state. This highlights the necessity of accounting for stochastic factors in predicting ecosystem dynamics. Moreover, the effect of time delays on system stability was investigated. It was found that significant delays in population responses can alter model dynamics, inducing more complex oscillation patterns or even chaotic behaviour. This is a crucial aspect to consider when modelling long-term ecological processes. An important practical implication of this study is the potential application of its findings in developing ecological management strategies. For instance, regulating species interaction parameters may help control population sizes and prevent excessive fluctuations that could lead to ecological instability. Thus, the study demonstrates that the Hopf bifurcation is a critical mechanism to consider in population dynamics modelling. Incorporating stochastic factors, time delays, and parameter variations significantly enhances the realism of mathematical models, making them valuable for ecological management applications (Kyfyak et al., 2024; Kerimkhulle et al., 2023). This is particularly relevant in light of the results of Sukarsih et al. (2023), who, using fuzzy logic and the numerical Runge-Kutta algorithm, showed that uncertainty in initial conditions and parameters can alter model behaviour, bringing it closer to realistic natural ecosystem conditions.

In accordance with the work of Yan et al. (2022), the system's behaviour is highly dependent on the presence of an open advective environment. The study confirms that additional ecological factors can modify the stability of stationary points and induce bifurcations, consistent with prior findings. As noted by Ye et al. (2019), the Allee effect can significantly influence predator-prey model dynamics by altering their phase structure. Although the model does not directly account for the Allee effect, analogous dynamic changes can be observed through modifications in the Holling type II functional response parameters, which may induce oscillatory regimes via Hopf bifurcation.

A third critical aspect is the influence of chaotic signals and stochastic perturbations, as examined in the work of Upadhyay and Kumari (2018). The model also exhibits complex nonlinear behaviour under parameter variations, which may be interpreted as manifestations of chaotic phenomena in real ecosystems. This could be significant in identifying unstable regimes in population dynamics. Furthermore, Los Huertos (2020) explores interspecies interactions extending beyond classical competition. The model also demonstrates nonlinear predator-prey interactions affecting their long-term evolution and potential coexistence scenarios. Thus, the study demonstrates that the Hopf bifurcation in a predator-prey model with a Holling type II functional response is a key mechanism governing long-term ecosystem dynamics. Future research may focus on investigating the effects of time delays, stochastic perturbations, and interspecies interactions on system stability.

4. CONCLUSION

Hopf bifurcation is a key process that propels predator-prey population dynamics from stable equilibria to oscillatory and chaotic regimes, as this work methodically showed. The study determined crucial parameter thresholds that control ecosystem stability and bifurcation type (supercritical vs. subcritical), specifically the predator-prey interaction coefficient, saturation level, predator mortality, and conversion efficiency, by combining a Holling type II functional response with time delays and stochastic perturbations. Clarifying how random fluctuations and temporal delays can cause ecosystems to become unstable by altering bifurcation behaviour and perhaps resulting in chaotic dynamics or unstable oscillations is the main contribution. By adding physiologically realistic elements like predator saturation and delayed responses, this nuanced understanding expands on traditional predator-prey models and offers a more precise forecasting framework for ecosystem behaviour under environmental unpredictability. These findings have two practical implications: resource managers and conservationists can use the determined bifurcation thresholds to create successful intervention strategies, and ecological researchers can use the findings to develop a strong analytical and numerical tool for forecasting important changes in population dynamics. To prevent destabilising oscillations and assist biodiversity conservation and ecological resilience in the face of environmental change, these could involve controlling predator mortality or regulating prey supply. Future studies should investigate the effects of climate change on bifurcation dynamics and extend this model to incorporate regional heterogeneity and multispecies interactions. The model's suitability for predicting ecosystem reactions and directing adaptive ecological management will be further improved by such advancements.

Conflict of Interest
The authors declare no conflicts of interest.

Author Contribution Statement
Dorina Guxholli: Conceptualization, investigation, writing-original draft preparation, methodology, and formal analysis. Valentina Shehu: Conceptualization, writing-original draft preparation, methodology, formal analysis, writing-review and editing, visualization, and supervision.

Data Availability Statement
The authors confirm that the data supporting the findings of this study are available within the article.

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