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Research Article

Adaptive Exponential Smoothing with Embedded Fuzzy Adjustment for Nonlinear Time Series Forecasting

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ABSTRACT

Nonlinear time series forecasting is one of the challenges to be handled in traditional exponential smoothing (ES) due to its fixed smoothing parameter. In this study, a novel Adaptive Exponential Smoothing with Embedded Fuzzy Adjustment (AES-FA) method is proposed to dynamically change its smoothing parameter based on data characteristics. This study aims to develop an adaptive exponential smoothing model with embedded fuzzy adjustment, in which the smoothing parameters are dynamically updated to capture nonlinear patterns in time series data; to validate and evaluate the performance of the proposed model against the traditional exponential smoothing method; and to apply the best-performing model to a real-world nonlinear dataset for improved forecasting accuracy. An empirical study is used to compare AES-FA to traditional ES methods, Exponential Brownian Motion and Neural Network for 52 weeks for 2021-2024. A fuzzy logic system is embedded into the ES which is Single Exponential Smoothing, Double Exponential Smoothing and Holt-Winters to dynamically adjust its smoothing parameter based on volatility, trend and seasonality. Results are validated using MAE and RMSE. Results show that AES-FA has a lower value of MAE and RMSE between traditional ES. AES-FA has more stable residuals even during abrupt changes. In conclusion, AES-FA has improved forecasting performance for nonlinear time series data and has potential for real-world applications like forecasting agricultural product prices.

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1. INTRODUCTION

Time series forecasting examines the past data in a time-ordered manner to detect underlying trends and make estimates for the future values at regular time intervals such as weeks, months, and years. However, in practice, time series data may have various complexities and challenges. Despite various challenges in time series forecasting, the aim of this research work is to concentrate on the nonlinear time series to obtain more accurate and trustworthy results in the forecasting process. Nonlinearity in time series means the relationship between the variables is not properly represented by a linear relationship and hence suggests the past values may affect the future values in a complicated way (Khazaeiathar & Schmalz, 2025).

Consequently, obtaining an accurate forecast will be more difficult due to abrupt fluctuations and shifts in the data caused by nonlinear patterns. The nonlinear characteristic occurring in time series can be attributed to the complex and unpredictable properties of the data itself, which makes basic linear analysis less effective. Real-world data, such as those occurring in manufacturing systems, childhood mortality rate, and stock market value, are generally nonlinear and noisy (Adeyinka & Muhajarine, 2020; Fatima & Rahimi, 2024). Under such circumstances, time series tend to be nonlinear and noisy with unpredictable variations, and uncertainties which cannot be measured using traditional statistical models and linear frameworks (Pal & Kar, 2022). Additionally, Goswami (2019) pointed out that nonlinear systems are known to sudden shifts between different behaviour due to small changes in parameters, which cannot be measured using traditional statistical measures such as means and variance.

In this study, it examined the dataset that contain a nonlinear characteristic which is a price of tomatoes and chillis on a weekly basis to provide a dependable method of forecasting. The price of tomatoes is determined by the supply, demand, and seasonality which results in nonlinear and irregular behaviour (Okere & Balyan, 2025). This problem becomes more complicated when the time series presents nonlinear characteristics together with seasonal variability, further complicating the accuracy of the forecasts (Dudek et al., 2021). Despite the popularity of linear prediction models in terms of simplicity of use, their capabilities are limited in handling nonlinear time series processes. Several nonlinear models were developed to overcome these limitations including ARCH and TAR (Liu et al., 2021). Besides these nonlinear techniques, some traditional statistical forecasting methods such as exponential smoothing can also be used in time series forecasting. However, their capabilities might be limited in handling very nonlinear processes in time series. Traditional forecasting techniques mainly depend on the assumption of linearity and hence fail in the complicated and realistic time series data. Additionally, the presence of nonlinearity, noise, and non-stationarity in the time series data of the real world has further affected the accuracy of the forecast (Wang & Liu, 2025). Valentina et al. (2024) point out that smoothing algorithms like Exponential Smoothing (ES) and Double Moving Average (DMA) have been designed with time series data where the trend patterns keep on changing continuously. Therefore, such algorithms have the potential to generate lagging signals and can only handle the linear relationship in the data.

Traditional forecasting techniques like exponential smoothing continue to be among the most widely used methods because of its simplicity and precision. Exponential smoothing methods, including Single Exponential Smoothing (SES), Double Exponential Smoothing (DES) and Holt's Winter are the simplest method to be used and effective in modelling of level, trend and seasonality (Lazim, 2014; Junthopas & Wongoutong ,2023; Permata et al., 2024). According to Aini et al. (2022), the Holt-Winters (HW) approach provides more accurate forecast data with lower error values. However, these approaches are based on linear assumptions and constant smoothing parameters (α, β, γ), which restrict their capability to process nonlinear and dynamic data (Lai et al., 2006). Dixon (2022) pointed out that if the smoothing parameter α is fixed, the method imposes a fixed pattern of partial autocorrelation, which limits the method to being applied to stationary or slowly moving time series.

Exponential smoothing has an assumption that the patterns in the future will remain as they were in the past. Hence, this is not always the case, especially when dealing with volatile environments such as stock markets (Sun & Deng, 2025). In cases where data is highly volatile or has complex nonlinearity patterns, models based on exponential smoothing may struggle to perform well even when data is seemingly structured properly. Moreover, according to Djakaria and Saleh (2021), traditional approaches such as the Holt Winters' exponential smoothing method require trial and error procedures to determine appropriate α, β, and γ settings to ensure effective model selection and minimize forecasting errors measured using metrics such as Mean Absolute Percentage Error (MAPE). This process is more challenging when dealing with non-linear and non-stationary situations where model parameters change over time, resulting the forecasts that are either inconsistent or suboptimal. Hence, many studies have hybrid exponential smoothing with other methods to handle nonlinear in time series.

One of them is a hybrid ES-dRnn model that was presented by Smyl et al. (2023) that combines exponential smoothing and a dilated neural network model to forecast short-term demand requirements. This hybrid model has been shown to outperform statistics and machine learning models within a case study of 35 European countries by identifying trends within linear and nonlinear patterns simultaneously.

Fuzzy Time Series was proposed by Song and Chissom (1993) to deal with imprecision and uncertainty associated with time series data. Numerous research efforts have demonstrated the efficiency of Fuzzy Time Series models for a wide array of applications, such as forecasting the tourism demand, energy load, stock market index analysis, and many additional real-world forecasting issues. Several studies have integrated fuzzy-based approaches with exponential smoothing to capture nonlinear patterns and improve forecasting performance (Bidin et al., 2022; Valentina et al., 2024; Hameed & Abbood, 2025;). In another study, the performance of Fuzzy Time Series is compared to the performance of the HW method for forecasting the number of tourist arrivals in Langkawi. Fauzi et al. (2020) revealed that the best method is the HW method. This show that exponential smoothing can still yield accurate forecast than Fuzzy Time Series, yet additional methods would be needed to enhance the accuracy when handling nonlinear time series. Given this variability of findings, there is a need for a dynamic mechanism to improve the effectiveness of exponential smoothing when facing nonlinear issues.

Fuzzy logic was originally developed by Zadeh (1965) using fuzzy set theory as a mathematical tool to deal with ambiguity and uncertainty in real-world problems. Unlike binary logic, fuzzy logic does not rely on fixed memberships of 0 or 1 but rather on continuous memberships between 0 and 1 to represent imprecise and non-linear systems. Besides hybrid exponential smoothing, there also a study that applies hybrid in fuzzy time series to handle nonlinear in time series such as study by Samanta (2021) that making the hybrid suitable for nonlinear time series by integrating data-driven learning with fuzzy-rule based reasoning. In this regard, this research aims to propose a novel Adaptive Exponential Smoothing with Embedded Fuzzy Adjustment (AES-FA) model to adjust the parameters of exponential smoothing (α, β, γ). A closely related study is by Bicen (2017) which utilized the fuzzy inference to adjust exponential smoothing parameters according to variance changes, particularly for handling outliers and level shifts. Nevertheless, this study mainly focuses on SES rather than the nonlinear in time series which by adaptively adjusting the smoothing parameter across multiple method in traditional exponential smoothing.

This research focuses on the limitations of the traditional exponential smoothing technique in relation to flexibility in the context of applying the technique to a nonlinear and volatile time series dataset. To address these limitations, an AES-FA is presented which aims to improve the forecasting results. The model is evaluated using nonlinear datasets, which is weekly tomato and chilli prices. Although well-established models are ARIMA and GARCH, this paper aims at improving exponential smoothing of nonlinear time series forecasting. Exponential Smoothing is often considered deterministic, it remains widely used because it is simple and has good short term forecasting ability. To overcome this weakness in processing nonlinear and volatile data, a fuzzy adjustment mechanism is added to enable adaptive parameter updating. Moreover, the AES-FA performance is further compared with traditional exponential smoothing, Stochastic benchmark (Exponential Brownian Motion) and Neural Network models to provide more comprehensive evaluation.

2. METHODOLOGY

This section introduces an AES-FA method. The main aim of this model is to improve the traditional exponential smoothing method by making its smoothing parameters adaptively change based on the dynamic behaviour of the data. The traditional exponential smoothing method uses fixed smoothing parameters and are very effective in stable time series but struggle to deal with non-linear patterns. The AES-FA system uses a fuzzy inference system to continuously modify the smoothing parameters (α, β, γ) which act as an adaptive controller with a self-adjusting mechanism and updated only when the prediction error exceeds a predefined threshold. Adaptive fuzzy forecasting models typically utilize threshold-based updating mechanisms, where the model's parameters are adjusted only when the prediction error exceeds a defined threshold, thereby avoiding unnecessary changes (Wang & Liu, 2025).

2.1. Research Framework

This section described the theoretical framework for conducting the research and how the study was developed. In short, the proposed model in focus was the adaptive exponential smoothing model. This model used fuzzy adjustments that aimed to adjust the smoothing parameter in the exponential smoothing model for the weekly prices of tomatoes and chillies. The theoretical framework for adaptive exponential smoothing, strengthened by the fuzzy model, was shown in Figure 1.

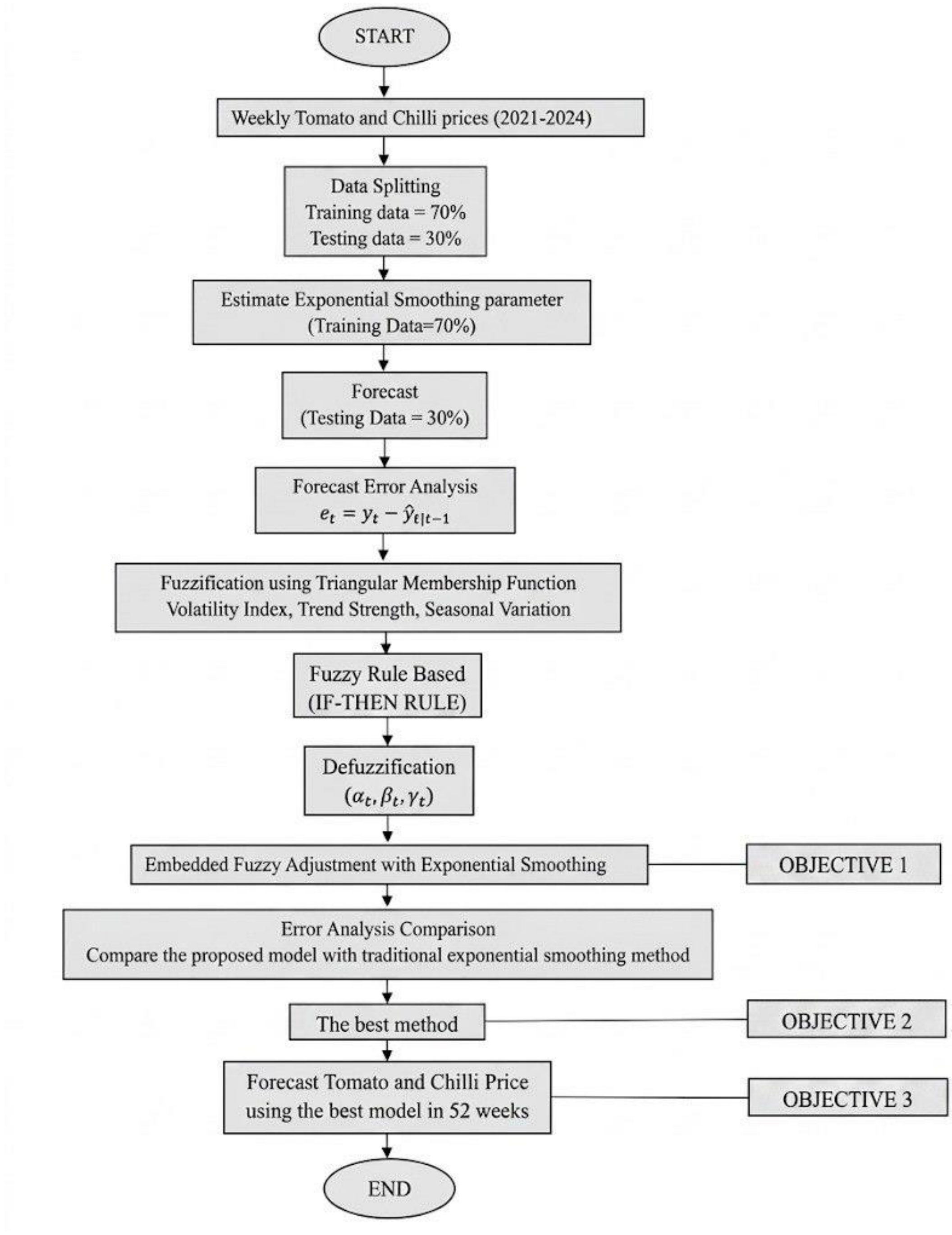

Figure 1. Conceptual framework of the proposed AES-FA model

2.2. Source of Data

The AES-FA Model forecasts the agricultural prices every week by using the actual price of tomatoes and chillis obtained from the Federal Agricultural Marketing Authority (FAMA) in Malaysia for the period of 2021-2024. The data contains 208 observations for a period of 52 weeks every year and it is appropriate for testing models because the dataset have a nonlinear characteristic. Firstly, the dataset is analysed using statistical testing via Brock-Dechert-Scheinkman (BDS) test to confirm the presence of nonlinearity in the data. Then, this dataset is split into a training and testing in a ratio of 70:30 chronologically using Microsoft Excel. The first 146 observations were used for training, and the remaining 62 observation were used for testing.

2.3. Estimate Exponential Smoothing Parameter

Exponential Smoothing was a traditional forecasting method generally applied for short-term forecasts of a univariate time series. In this study, the most appropriate traditional models of Exponential Smoothing, namely Single Exponential Smoothing (SES), Double Exponential Smoothing (DES), and Holt-Winters (HW), were applied. Winters (1960) contributed to the development of the HW method, while Brown (1959) presented SES as a simple yet effective method for short-term forecasting. SES was considered appropriate for a series with no trend and no seasonality, as it focused on the level component by combining past data with forecasted values. The equation for SES was expressed as Equation (1).

$$l_t = \alpha y_t + (1-\alpha)l_{t-1} \tag{1}$$

where l_t is the level component at time t, y_t is the actual value at time t, l_{t-1} is the previous smoothed level and the smoothing parameter, α is determined with values between 0 and 1, that controls weight to the current observation and the past forecast.

SES is not suited for time series data that has a trend. In a bid to overcome this drawback, Holt (1957) had modified SES to allow for data with a trend. This improvement is known as Holt's linear method or DES and the equation are as follows Equation (2) and (3).

$$l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1}) \tag{2}$$

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} \tag{3}$$

where b_t is the estimated trend component and β is the trend smoothing parameter to be determined with values between 0 and 1. Then, Winters (1960) improved the Holt's method to capture seasonality. This technique is called the HW method or Triple Exponential Smoothing, which extends Holt's method to incorporate an extra component which is seasonal component and the equation are as follows as Equation (4) - (6).

$$l_t = \alpha \frac{y_t}{s_{t-m}} + (1-\alpha)(l_{t-1} + b_{t-1}) \tag{4}$$

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} \tag{5}$$

$$s_t = \gamma \frac{y_t}{l_t} + (1-\gamma)s_{t-m} \tag{6}$$

where γ is the smoothing parameter for seasonality, 0< γ<1, m is the length of seasonality and h is the number of steps ahead to be forecast. Unlike exponential smoothing, where the values of alpha (α), beta (β), and gamma (γ) are fixed, in the AES-FA proposed model, the values are determined through fuzzy logic at each time step.

2.4. Forecast Testing Data

The forecasting was conducted on the testing data, which represented 30% of the dataset, to evaluate the performance of the model. Forecasts for one-step ahead and multi-step ahead were generated based on the respective exponential smoothing methods.

Single Exponential Smoothing (SES): $$\hat{y}_{t+1} = l_t \tag{7}$$

Double Exponential Smoothing (DES): $$\hat{y}_{t+h} = l_t + hb_t \tag{8}$$

Holt-Winters (HW): $$\hat{y}_{t+h|t} = (l_t + hb_t)s_{t-m+h} \tag{9}$$

2.5. Forecast Error Analysis

The fuzzy adjustment system uses these dynamic forecast errors and trend changes as input. The one-step ahead error is defined as Equation (10).

$$e_t = y_t - \hat{y}_{t|t-1} \tag{10}$$

2.6. Fuzzification using Triangular Membership Function

This model proposes an Adaptive Parameter Estimation through Fuzzy Adjustment, which uses a fuzzy inference system (FIS) to adjust the parameters (α, β, γ) based on characteristics found in a time series. The fuzzy adjustment process is driven by the forecasting error, and a larger error indicate the need for stronger corrective updates (Soambaton & Nugroho, 2025). This matches fuzzy control theory, which strengthens control with an increase in error. Since the smoothing parameters in Exponential Smoothing usually lie in the interval [0,1], and since these parameters can affect forecasting accuracy (Handika & Satwika, 2023), in the AES-FA model, the fuzzy adjustment process will be activated only if the forecasting error exceeds a certain threshold (error >1), thus indicating a large difference between the actual and forecasted values.

2.6.1. Input Variable Definition

Three input variables will be conducted which is volatility index that represents short-term instability of the series, trend strength that captures the persistence of directional changes in the data and seasonal variation that measured using the dispersion of the seasonal components generated by the HW method and high variation indicates high seasonality. The formula for each input variables as Equation (11-16).

$$v_t = \min\left(1, \frac{|e_t|}{c_e}\right) \tag{11}$$

$$c_e = median(|e_{t-q+1}|, \ldots., |e_t|) \tag{12}$$

$$t_t = \min\left(1, \frac{|b_{t-1}|}{c_b}\right) \tag{13}$$

$$c_b = median(|b_{t-q+1}|, \ldots., |b_t|) \tag{14}$$

$$S_t = \min\left(1, \frac{|s_{t-1}|}{c_s}\right) \tag{15}$$

$$c_s = median(|s_{t-m-q+1}|, \ldots., |s_{t-m}|) \tag{16}$$

where c_e is a robust scale parameter that is derived from the median absolute deviation of past residuals. As v_t increases, it indicates large variations and abrupt changes. The constant c_b and c_s are derived by normalization with reference to the median of past values of trend and seasonal magnitudes. The three inputs are normalized within [0, 1] and serve as linguistic inputs for fuzzification.

2.6.2. Fuzzification Stage

In this phase, all input variables are represented in fuzzy sets. For each input in the dataset, the fuzzy relationship is described using a fuzzy number. This expresses how well this input is in the fuzzy set using membership functions (Chen, 1996). There are several common forms for membership functions in fuzzy systems, with trapezoidal, triangular, and Gaussian being the most frequently used (Lucas et al., 2022). Hence, this study applies a triangular membership function for converting each input variable into its fuzzy set as they provide a comprehensible means to denote linguistic variables and are widely employed in fuzzy time series forecasting methods (Wang & Liu, 2025). Each input variable is represented through three triangular membership functions where for volatility index and seasonal variation are low, medium and high, and for trend strength are weak, moderate and strong. The membership functions are defining as follows as Equation (17) - (19).

$$\mu_{low/weak}(x) = \max(0, 1 - 2x) \tag{17}$$

$$\mu_{medium/moderate}(x) = 1 - |2\mathrm{x} - 1| \tag{18}$$

$$\mu_{high/strong}(x) = \max(0, 2\mathrm{x} - 1) \tag{19}$$

The variable x is bounded within the normalized interval [0,1]. The use of triangular functions enables linguistic states to change smoothly, thus promoting adaptation capabilities of the fuzzy system when handling the time series.

2.7. Fuzzy Rule Based

There are three common approaches of creating fuzzy IF-THEN rule which is expert-drive (heuristic), data-driven and hybrid methods which is the combination of expert knowledge and data learning. The rule base used in this work was developed through a heuristic expert-based way by which the linguistic connections among volatility, strength of trends and seasonal variation are converted into time-series behaviour IF-THEN rules. Specifically, a significant shift in the time series results in increased volatility, and thus larger smoothing parameters are needed to support quick adaptation, so that, when the changes are small, volatility is low, so the updates to smoothing are also small. Additionally, trends with strong trends will need larger trend smoothing factors and higher seasonal variation needs larger seasonal smoothing changes. Table 1 displays the complete set of fuzzy rules. Once the fuzzy rule base developed, the linguistic rules are analysed to find their level of activation for given conditions. Fuzzified inputs are processed by FIS with the help of the fuzzy rule base to produce fuzzy outputs. The inference mechanism analyses levels of rule activations to combine fuzzified inputs with the fuzzy rule base.

Table 1. Fuzzy rule based for adaptive smoothing parameter

Rule	Fuzzy Statement	Rule	Fuzzy Statement
R1	IF volatility is Low THEN α is Small	R6	IF trend is Strong THEN β is Large
R2	IF volatility is Medium THEN α is Medium	R7	IF seasonal variation is Low THEN γ is Small
R3	IF volatility is High THEN α is Large	R8	IF seasonal variation is Medium THEN γ is Medium
R4	IF trend is Weak THEN β is Small	R9	IF seasonal variation is High THEN γ is Large
R5	IF trend is Moderate THEN β is Medium		

2.8. Defuzzification

After obtaining fuzzy outputs from the FIS, a defuzzification process is required to ensure that the fuzzy output is given a specific numerical value for effective implementation (Mamdani & Assilian, 1975). The Mamdani type of inference process is used to combine the outputs of the rules. The fuzzy outputs for each parameter are defuzzified using the centroid method as Equation (20).

$$\theta_t = \frac{\int \mu_\theta(x)x dx}{\int \mu_\theta(x)dx}, \quad \theta \in \{\alpha, \beta, \gamma\} \tag{20}$$

To avoid stability issues and fluctuations in the values of the defuzzified parameters for the smoothing function, these parameters must remain within the defined limits 0 < a_t, β_t, γ_t< 1.

2.9. Embedded Fuzzy Adjustment with Exponential Smoothing

The adaptive forecasting technique used by the AES-FA model combines the basic exponential smoothing model and utilizes a fuzzy feedback system to refine the values of the smoothing factors for each time step. For the model to be adaptable, the fuzzy adjustment technique will be activated when the error value surpasses a certain threshold. In other words, when the error value is higher than 1, the fuzzy feedback technique will be applied to modify the values of the smoothing parameter. The new values of the smoothing parameters (a_t, β_t, γ_t) are then used to calculate the level, trend, and seasonal components of the exponential smoothing equations to generate the h-step-ahead forecasts.

2.10. Error Analysis Comparison

A standard error measures are used to evaluate the forecasting accuracy. The equation of Root Mean Square Error (RMSE) as Equation (21). RMSE penalizes large errors more heavily and will provide a well-balanced perspective on model performance.

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_t - \hat{y}_{t|t-1})^2} \tag{21}$$

2.11. Benchmark Models

In order to analyze the effectiveness of the proposed AES-FA model, several benchmarks' models are applied for comparison. These consist of a stochastic model, namely Exponential Brownian Motion and a data-driven model, namely Neural Network. The purpose of including these models is to conduct a comprehensive analysis of the efficacy of the proposed models.

2.11.1. Exponential Brownian Motion (EBM)

EBM is used as a benchmark stochastic model to describe random variations in price series. This stochastic process assumes that price fluctuations follow a continuous time stochastic process involving a deterministic drift component as well as a stochastic volatility component. The price process can be described by the following Equation (22):

$$dS_t = \mu S_t dt + \sigma S_t dB_t \tag{22}$$

where S_t denotes the price at time t, μ represents the drift term, σ is the volatility, and B_t is a Wierner process. The model suggests that the price fluctuations are independent and follow to a lognormal distribution, which makes it appropriate for modelling financial and commodity price patterns (Salaudeen et al., 2023).

2.11.2. Artificial Neural Network (ANNs)

ANNs are commonly used in time series forecasting due to the ability to handle complex non-linear relationships without requiring any assumption about the internal structure of the data (Bucci, 2020). Unlike traditional statistical models, ANNs can efficiently capture hidden patterns, trends, and irregular in nonlinear data, hence making it suitable for real-world forecasting applications. In this study, a neural network model is used as a benchmark. The model is trained on historical prices, where lagged prices are used as input variables and the output corresponds to the forecasted prices. The proposed model uses a data-driven approach, where the model is learned directly from the data without any prior assumptions. The performance of the neural network model is evaluated by using accuracy measures which is MAE and RMSE.

2.12. Computational Procedures

For more insight of the proposed AES-FA model, a numerical example is given to show the computations for updating the smoothing parameter: level, trend and seasonal components. The following weeks were calculated using an iterative approach based on the same process.

Step 1: Parameter estimation and exponential smoothing component updating using SES. The smoothing parameter α for SES is estimate using the training dataset (70%) in Microsoft Excel. Assume that the obtained value is $\alpha = 0.614357$. The smoothed level is update using Equation (1). Initialization:

$$l_1 = y_1 = 2.55 \tag{23}$$

Level at week 2: $$l_2 = 0.6144(6.32) + (1 - 0.6144)(2.55) = 4.8661 \tag{24}$$

Step 2: Forecast and error computation. For SES, the one-step-ahead forecast is:

$$\hat{y}_{t+1} = l_t \tag{25}$$

Hence, the forecast for week 3 is: $$\hat{y}_3 = l_2 = 4.8661 \tag{26}$$

The error at week 3 is: $$e_3 = 6.55 - 4.8661 = 1.6839 \tag{27}$$

If the absolute value of the forecast error is greater than 1, then the fuzzy adjustment module is triggered to change the value of the smoothing parameter. But if it is less than 1, the fuzzy module is not triggered, and the same smoothing parameter α is used. Since $|e_3| = 1.683876 > 1$, fuzzy adjustment is activated.

Step 3: Fuzzification phase. Based on Equation (12) with q=10, the scaling factor c_e is compute from the median of the last 10 absolute error:

$$c_e = median(|e_{t-9}|, |e_{t-8}|, \dots, |e_t|) \tag{28}$$

For instance, assume the $c_e = 1.01$. By using Equation (11), the volatility index is computed as:

$$v_t = \min(1, \tfrac{1.6839}{1.01}) = \min(1, 1.6672) = 1 \tag{29}$$

Then, the volatility index is fuzzified into three linguistic form which is low, medium and high using triangular membership function using Equation (17)- Equation (19).

$$\mu_{low}(1) = \max(0, 1 - 2(1)) = 0 \tag{30}$$

$$\mu_{medium}(1) = 1 - |2(1) - 1| = 0 \quad (31)$$

$$\mu_{high}(1) = \max(\, 0{,}2(1) - 1) = 1 \quad (32)$$

Step 4: Defuzzification. Since the membership degrees for volatility are $\mu_{low}(1) = 0$ and $\mu_{medium}(1) = 0$, rule R1 and R2 in Table 1 are inactive. Meanwhile $\mu_{medium} = 1$ is activated and the fuzzy output corresponds to α is large. The defuzzification process is carried out using the centroid method in Equation (20) to obtain a crisp value of the adaptive smoothing parameter, α, ranging between 0 and 1. In this study, the output fuzzy sets for α are within the bounds in Equation (33-35).

$$\alpha_{small} : [0{,}0.5] \quad (33)$$

$$\alpha_{medium} : [0.25{,}0.75] \quad (34)$$

$$\alpha_{large} : [0.5{,}1.0] \quad (35)$$

Since only R3 is activated, the aggregated output membership function becomes $\mu_{\alpha}(x) = \mu_{Large}(x)$. Therefore, the defuzzifed value is compute. Thus, the defuzzified adaptive smoothing parameter is 0.75.

$$\alpha_t = \frac{\frac{1^2 - 0.5^2}{2}}{1 - 0.5} = 0.75 \quad (36)$$

Step 5: Embedded Fuzzy Adjustment with Exponential Smoothing. Next, the Equation (1) is update using new parameter obtain in Equation (36). The updated level at week 3 is in Equation (37).

$$l_3 = 0.75(6.55) + (1 - 0.75)(4.8661) = 6.1290 \quad (37)$$

Since the one-step-ahead SES is based on the level component, the AES-FA forecast for week 4 is 6.1290. A similar embedded fuzzy adjustment technique is used in all exponential smoothing models. For DES and the HW method, the defuzzified adaptive variables are incorporated into their update equations for dynamic updates of level, trend, and seasonality at each step. This process is repeated across the iterations to obtain the final forecasts using AES-FA.

3. RESULTS AND DISCUSSION

This section presents the outcome of the forecasting process obtained using the weekly price of tomatoes and chilli data analysed using Microsoft Excel. The AES-FA approach is used by incorporating a fuzzy adjustment component to adjust the parameters in the exponential smoothing method if the error exceeds a predefined threshold. The main aim is to evaluate if the proposed model can result in enhanced accuracy compared to the traditional exponential smoothing by using a dataset that contain nonlinearity characteristics. Ultimately, the best model will be applied for forecasting the weekly price of tomato and chilli for the year 2025.

3.1. Preliminary Analysis of Data

In this section, preliminary analysis is carried out to discuss the features of the tomato and chilli price series provided by Brock, Dechert, and Scheinkman test, denoted as BDS test and the time series plots. The BDS test is used to determine the existence of nonlinear dependence, and the null hypothesis states that the data are independently and identically distributed, denoted as i.i.d. The null hypothesis is rejected, which means that there is nonlinearity in the series (Broock et al., 1996). The findings indicated that the p-value of tomato and chilli price series are both equal to 0.00 and this is lower than the usual level of significance of 0.05. Thus, the null hypothesis is rejected and it implies that these two datasets have a significant nonlinear characteristic. To deeper verify this finding, visual analysis of the data is conducted using time series graphs. Figure 2 presents the train-test partition of tomato prices, while Figure 3 shows the corresponding plot for Chilli prices. These plots represent the actual price data, divided into training and testing sets for model development and evaluation. Both series display noticeable fluctuations and irregular patterns over time, further supporting the presence of nonlinearity.

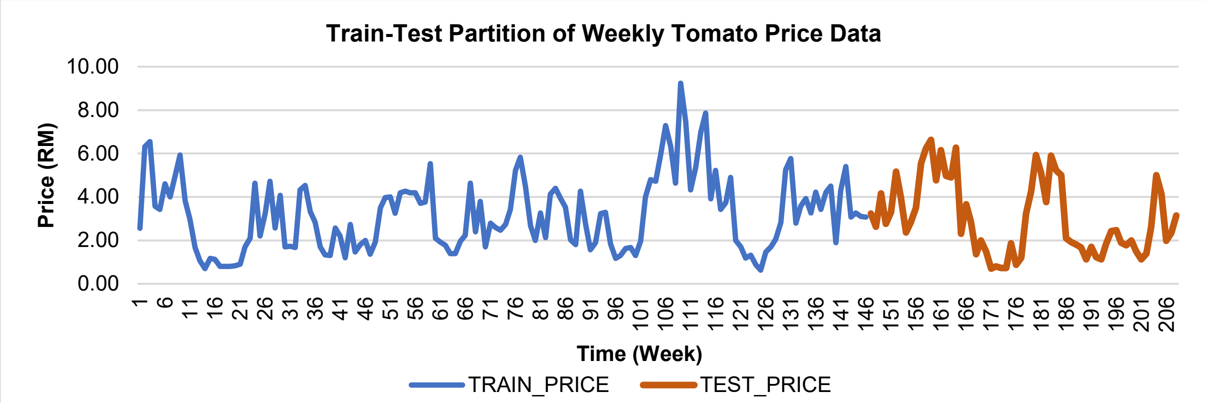

Figure 2. Train-Test Partition of Weekly Tomato Price Data

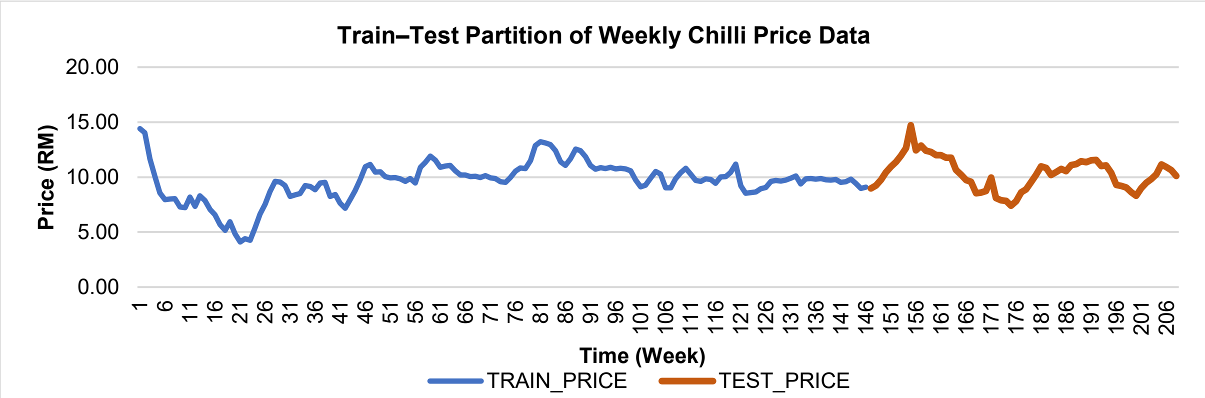

Figure 3. Train-Test Partition of Weekly Chilli Price Data

3.2. Comparison of Training and Testing Performance

In this section, the forecasting performance of the baseline models is evaluated in terms of training and testing RMSE. The training is used to gauge the goodness of fit of the model to the training data whereas the testing is used to gauge the predictive power of the model on unknown data. The findings indicate that SES is more effective with lower RMSE values when using tomato prices and DES when using chilli prices compared to the rest of the baseline models (Table 2). The general values of RMSE, however, are quite large which means that these models do not necessarily represent the data patterns behind them. This gives a reason to adopt the use of fuzzy-adjusted models, which will be anticipated to enhance the quality of forecasting in the next subsection.

Table 2. Comparison of training and testing performance for Tomato and Chilli Price Forecasting Models

Model	Tomato		Chilli	
	Training RMSE	**Testing RMSE**	**Training RMSE**	**Testing RMSE**
Single Exponential Smoothing	1.3159	1.2025	0.7707	0.7476
Double Exponential Smoothing	1.5515	1.2860	0.6863	0.7060
Holt-Winters	1.9582	1.6771	1.7284	1.0564

3.3. Traditional Exponential Smoothing and AES-FA Model

In this section, the performance of traditional exponential smoothing methods for modelling series on the test data set, using RMSE as evaluation criteria, will be assessed for SES, DES, and the HW method. As shown in Table 3, SES has the lowest RMSE for the tomato price while DES has the lowest value of RMSE for the chilli price. To evaluate the effect of fuzzy parameter adaptation, the fuzzy adjustment process is incorporated into each of the exponential smoothing techniques, resulting in the development of the single ES-FA, double ES-FA, and HW ES-FA.

Table 3. Performance of the Traditional Exponential Smoothing on Tomato Price and Chilli Price

Model	Tomato Price	Chilli Price
	RMSE	**RMSE**
Single Exponential Smoothing	**1.2025**	0.7476
Double Exponential Smoothing	1.2860	**0.7060**
Holt-Winters Exponential Smoothing	1.6771	1.0564

Table 4 shows that the AES-FA model, developed by integrating the fuzzy adjustment process, improves the accuracy of all the techniques, as measured by the reduction in MAE and RMSE values, compared to their standalone exponential smoothing. In particular, the Holt- Winters ES-FA shows the best performance on tomato Price, with the lowest RMSE value of 0.2675, while Double ES-FA shows the best performance on Chilli Price, with the lowest RMSE value of 0.3097. This outcome shows that the fuzzy adjustment in the embedded model can address the issue of nonlinear in time series by adjusting the smoothing parameters dynamically whenever the forecast error surpasses a certain level. This outcome ensures the fulfilment of the research objective, which is to improve the basic exponential smoothing methods for nonlinear time series forecasting.

Table 4. Performance of the Adaptive Exponential Smoothing with embedded Fuzzy Adjustment on Tomato Price and Chilli Price

Model AES-FA	Tomato Price	Chilli Price
	RMSE	RMSE
Single ES - FA	1.1973	0.7476
Double ES - FA	0.5332	**0.3097**
Holt-Winters ES - FA	**0.2675**	0.3701

3.4. Benchmark Comparison

This section will include the comparative analysis of proposed AES-FA model and the selected benchmark models such as Exponential Smoothing, Exponential Brownian Motion, and Neural Network. The comparison is based on Root Mean Square Error (RMSE), and the lower the values, the higher the forecasting accuracy. Based on Table 5, AES-FA model gives the lowest values of RMSE compared to other models for the tomato and chilli prices. Overall, the findings clearly indicated that the proposed AES-FA model performs better than all benchmark models in both tomato and chilli price series. This underscores the efficiency of the integration of fuzzy adjustment in improving the forecasting accuracy, particularly in handling nonlinear patterns in the data.

Table 5. Comparative Analysis of AES–FA and Benchmark Models for Forecasting Tomato Prices and Chilli Prices

Model	Tomato Prices	Chilli Prices
	RMSE	RMSE
AES-FA	**0.2675**	**0.3097**
Holt-Winters	1.6771	0.7060
Exponential Brownian Motion	2.5134	2.2858
Neural Network	1.8546	3.1346

3.5. Tomato and Chilli Price Forecast for the Year 2025

3.5.1. Forecast for Tomato Price

In this section, the proposed model of AES-FA by applying in HW model as baseline exponential smoothing is used to generate a forecast of tomato prices in the year 2025. Figure 4 shows the actual price from 2021 to 2024 and 52-week ahead forecast of tomato price in 2025 using the proposed AES-FA approach.

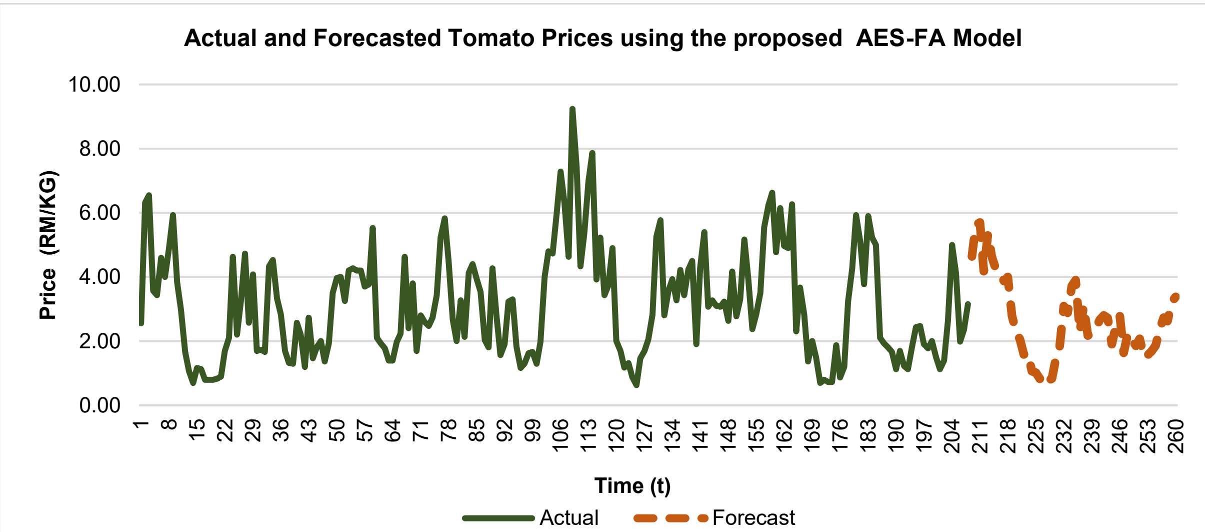

Figure 4. Actual data and 52-Week Ahead Forecast for the Year 2025 of Tomato Price using AES-FA

Overall, the forecast clearly shows that the price fluctuates throughout the year, as it experiences a peak in the initial months of the year, a drop in the middle of the year, and a gradual increase in the final months of the year. To enable a better numerical analysis of the forecast in Figure 4, the values of the forecast for each week are given in Table 6. As summarised in Table 6, the highest forecasted price is RM 5.70/kg in Week 3, while the lowest is RM 0.75/kg in Week 20.

Table 6. Weekly Forecasted Tomato Prices for 2025 (AES-FA)

Week	Price	Week	Price	Week	Price	Week	Price	Week	Price
1	4.64	12	2.32	23	1.84	34	2.80	45	1.56
2	5.62	13	2.06	24	3.09	35	2.72	46	1.69
3	5.70	14	1.58	25	2.88	36	1.90	47	1.86
4	4.18	15	1.64	26	3.72	37	2.44	48	2.35
5	5.35	16	1.07	27	3.90	38	2.78	49	2.73
6	4.63	17	1.01	28	2.30	39	1.63	50	2.63
7	4.24	18	0.83	29	3.10	40	2.25	51	3.22
8	4.16	19	0.96	30	2.18	41	2.07	52	3.38
9	3.87	20	0.75	31	2.23	42	1.85		
10	4.00	21	0.83	32	2.38	43	2.08		
11	2.80	22	1.45	33	2.66	44	1.50		

3.5.2. Forecast for Chilli Price

In this section, the proposed model of AES-FA by applying in DES model as baseline exponential smoothing is used to generate a forecast of chilli prices in the year 2025. Figure 5 shows the actual price from 2021 to 2024 and 52-week ahead forecast of chilli price in 2025 using the proposed AES-FA approach.

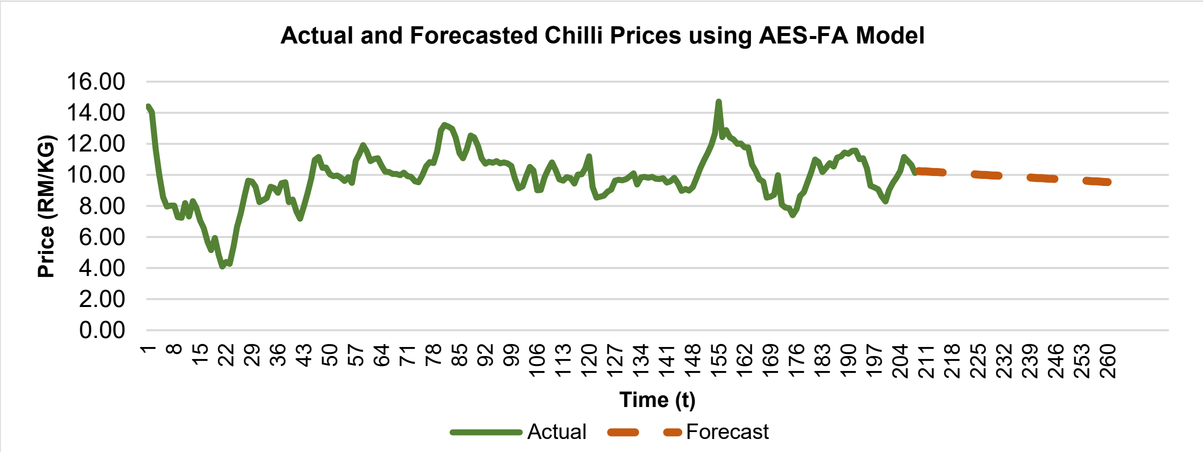

Figure 5. Actual Data and 52-Week Ahead Forecast for the Year 2025 of Chilli Price using AES-FA

The predicted prices of the chilli have a continuous and consistent downward trend throughout the 52 weeks horizon with the price figures declining by RM10.24 to RM9.54. The fact that there are no sharp changes indicates that the DES-FA model has been able to absorb the underlying trending part, which means that the predictions are constant and smooth. To enable a better numerical analysis of the forecast in Figure 5, the values of the forecast for each week are given in Table 7. As summarised in Table 7, the highest forecasted price is RM10.24/kg in Week 1, while the lowest is RM9.54/kg in week 52. Overall, this forecast offers a complete yearly view and could assist market stakeholders in strategizing for supply and pricing decisions.

Table 7. Weekly Forecasted Chilli Prices for 2025 (AES-FA)

Week	Price	Week	Price	Week	Price	Week	Price	Week	Price
1	10.24	12	10.09	23	9.94	34	9.79	45	9.63
2	10.23	13	10.08	24	9.93	35	9.77	46	9.62
3	10.22	14	10.06	25	9.91	36	9.76	47	9.61
4	10.20	15	10.05	26	9.90	37	9.75	48	9.59
5	10.19	16	10.04	27	9.88	38	9.73	49	9.58
6	10.18	17	10.02	28	9.87	39	9.72	50	9.57
7	10.16	18	10.01	29	9.86	40	9.70	51	9.55
8	10.15	19	10.00	30	9.84	41	9.69	52	9.54
9	10.13	20	9.98	31	9.83	42	9.68		
10	10.12	21	9.97	32	9.81	43	9.66		
11	10.11	22	9.95	33	9.80	44	9.65		

4. CONCLUSION

In conclusion, this paper examined the proposed AES-FA approach to forecast the weekly prices for tomato and chilli in Malaysia from 2021 to 2024 (Week 1-52) which contains a nonlinear characteristic. The presence of nonlinearity in the data was verified by using the BDS test and was confirmed visually from the time series plots. Tomato and chili prices were selected due to their practical relevance in Malaysia, where these commodities are widely consumed and exhibit highly volatile and nonlinear price behaviour that is dependant on supply and demand conditions as well as seasonality. However, the AES-FA proposed model is applicable to other time series data sets with nonlinear characteristics. As the data shows some non-linear variations, the approaches to be considered for forecasting are SES, DES, and the HW approach to understand the baseline performance. The findings indicate that traditional exponential smoothing may encounter constraints in forecasting nonlinear time series due to their dependence on fixed smoothing parameter. To overcome this situation, the fuzzy adjustment was incorporated for each respective smoothing method, allowing the fuzzy to adjust when the error threshold exceeded a predetermined level (error >1). Overall, the fuzzy-improved models performed better than the basic models, exhibiting improved RMSE, an indication that proposed AES-FA model can enhance forecasting performance. Based on the options explored, AES-FA models show the best performance with the lowest values RMSE, therefore was selected as the final AES-FA model for forecasting prices for tomato and chilli for the year 2025. The future work will include further improvement of the proposed AES-FA framework with better optimization of the adaptive fuzzy adjustment process as well as its robustness for the nonlinear time series forecasting task. Specifically, the optimization process for the membership function parameters of the fuzzy models can be accomplished automatically with neuro-fuzzy optimization techniques. Finally, the proposed model needs to be tested with real-world data with larger size, for instance, the multi-commodity FAMA data with one-week intervals.

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CONFLICT OF INTEREST
The authors declare no conflicts of interest.

AUTHOR CONTRIBUTION
Nur Hidayah Binti Ismail: Conceptualization, Methodology, Software, Writing original draft. Nur Amalina Binti Shafie: Supervision, Reviewing and Editing, Validation. Zahari Bin Md Rodzi: Supervision, Reviewing and Editing, Validation. Shaiful Anam. Reviewing and Editing, Validation.

DATA AVAILABILITY
The empirical validation uses data on the price of vegetables on a weekly basis for the period of 2021 to 2024. This data has been collected from the Federal Agricultural Marketing Authority (FAMA) and is available on the official FAMA portal. This data includes the price of tomatoes on a weekly basis, which shows seasonality and sudden changes. This data is available on the official FAMA portal and can also be requested through the authors.

DECLARATION OF GENERATIVE AI
During the preparation of this work, ChatGPT (OpenAI) was used to assist in language improvement, restructuring, and formatting of the manuscript. After using this tool, the content has been reviewed and edited as needed and the author will take full responsibility for the content of the published article.

ETHICS
Not applicable.

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