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Communicating Mathematically and Self-Regulating Strategies in Learning Mathematics

Berkomunikasi secara Matematik dan Strategi Terarah Kendiri dalam Pembelajaran Matematik

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Abstract

The purpose of this study was to investigate students' interactions in learning mathematics and their engagement with the self-regulated learning (SRL) strategies, particularly the cognitive learning strategies. The study involved a group of Year 9 students in one secondary school in the East of England engaged in mathematical tasks. This study employed the observational approach using video-recording as its primary method. Observational notes were kept and students' written work were taken into account to complement the video data. Powell *et al.*'s analytical model was used to analyse the videos. Events that were critical and significant to the study were fully transcribed. These critical events were analysed to examine students' interactions and their engagement with the cognitive learning strategies. In doing so, this study employed the mathematical communication framework, a combination of Sfard and Kieran's discourse analysis and Pintrich's SRL strategies approach. The findings showed that students were involved in an effective and productive discourse and were engaged with the components of the cognitive learning strategies.

Keywords self-regulating strategies, cognitive learning strategies, discourse analysis, secondary mathematics

Abstrak

Tujuan kajian ini untuk menyelidik interaksi murid dalam pembelajaran matematik dan penglibatan mereka dengan strategi terarah kendiri (STK), khususnya strategi pembelajaran kognitif. Kajian ini melibatkan sekumpulan murid Tahun 9 dalam melaksanakan tugasan matematik di salah sebuah sekolah menengah di Timur England. Kajian ini menggunakan pendekatan pemerhatian dengan rakaman video sebagai kaedah utamanya. Disamping itu, nota-nota pemerhatian telah direkodkan dan hasil kerja murid telah dikutip untuk melengkapkan data video. Model analitikal Powell *et al*. telah digunakan untuk menganalisis rakaman video. Peristiwa-peristiwa yang kritikal dan signifikan untuk kajian ini telah ditrankripsikan sepenuhnya. Peristiwa-peristiwa kritikal ini telah dianalisis untuk menyelidik interaksi murid dalam pembelajaran matematik dan kaitannya dengan strategi pembelajaran kognitif. Analisis telah dijalankan menggunakan kerangka komunikasi matematik iaitu satu kerangka yang menggabungkan antara analisis diskusi Sfard and Kieran dan pendekatan strategi STK Pintrich. Dapatan kajian menunjukkan murid-murid terlibat dalam diskusi yang produktif dan efektif, dan melibatkan diri dengan komponen-komponen strategi pembelajaran kognitif.

Kata Kunci Strategi pembelajaran kendiri, strategi pembelajaran kognitif, analisis diskusi, matematik sekolah menengah

Introduction

The study investigated students' self-regulating learning (SRL) strategies while engaging in mathematical tasks. Many studies have been carried out concerning mathematical problem solving processes, heuristics, and strategies but there have been only a few studies examining the effect of SRL strategies such as cognitive learning strategies, metacognitive and self-regulatory strategies, and resource management strategies on problem solving in mathematics (Pintrich, 1999).

The study also looked at students' interactions while engaging with the problems. Researchers in mathematics education agree, "that mathematics can and should, at least partly, be learned through conversation" (Ryve, 2004, p. 157). Communication has been observed as an essential element in mathematics teaching and learning (NCTM, 2000). NCTM (2000) outlines that a learner who has opportunities to engage in mathematical communication including speaking, reading, writing, and listening profits from two different aspects, communicating to learn mathematics and learning to communicate mathematically.

On the whole, this investigation linked the students' SRL strategies with their communication. This particular study would discuss students' interactions and their engagement with the SRL strategies, particularly the cognitive learning strategies in an attempt to observe a group of students working on mathematical tasks. Hence the research questions were formulated as follows:

1. Are students interacting with one another? If yes, what are the types of interactions?
2. In the course of these interactions, what SRL cognitive learning strategies do the students engage with?

Theoretical background

The mathematical communication framework developed for this study was designed to investigate and examine students' interactions and students' SRL strategies, particularly cognitive learning strategies in the context of a group solving mathematical tasks. This two-dimensional theoretical framework is a combination of Sfard and Kieran's (2001) discourse analysis framework and Pintrich's (1999) SRL model, particularly the component of cognitive learning strategies. The mathematical communication framework is shown in Figure 1.

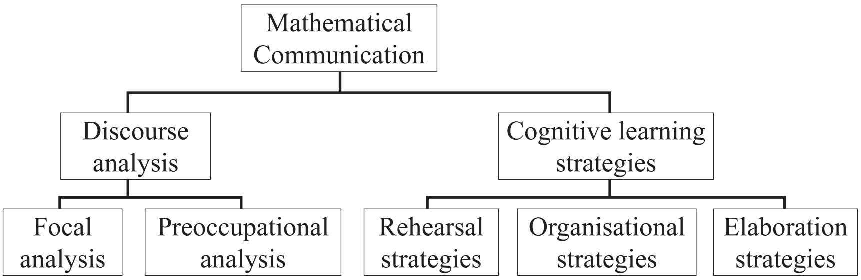

Figure 1 The mathematical communication framework

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Discourse Analysis: Sfard and Kieran (2001) develop a theoretical and methodological framework "which aims at characterising the students' mathematical discourses while they are working in groups" (Ryve 2006, p. 191). This framework, which is also known as communicational approach to cognition provides the platform to examine the efficiency and productivity of mathematical discourses. On the issue of effectiveness of communication, Sfard and Kieran observe that:

> The communication will not be regarded as effective unless, at any given moment, all the participants seem to know what they are talking about and feel confident that all the parties involved refer to the same things when using the same words.
>
> (Sfard & Kieran 2001, p. 51)

In examining the elements of effective and productive mathematical discourses, the framework offers two types of analyses: *focal* analysis and *preoccupational* analysis. On one hand, focal analysis deals with communicative successes or failures with no reasons revealed. On the other hand, preoccupational analysis offers the reasons behind the success or failure of a communication. Sfard and Kieran notice that:

> Focal analysis gives us a detailed picture of the students' conversation on the level of its immediate mathematical contents and makes it possible to assess the effectiveness of communication. This is complemented by preoccupational analysis, which is directed at meta-messages and examines participants' engagement in the conversation, thus possibly highlighting at least some of the reasons for communication failure.
>
> (Sfard & Kieran 2001, p. 42)

SRL strategies: The inclusion of the cognitive learning strategies as a component in the mathematical communication framework is to examine students' mathematical learning in the classroom. The cognitive learning strategies consist of three elements: rehearsal strategies, elaboration strategies, and organisational strategies. These strategies are important cognitive strategies associated with academic performance in the classroom whereby learners use them to control their cognition and learning [(McKeachie, Pintrich, Lin & Smith, 1986; Pintrich, 1989; Pintrich & DeGroot, 1990, in Pintrich, 1999)]. Artzt and Armour-Thomas (1992) state that students' cognitive behaviours "can be exhibited by verbal or nonverbal actions that indicate actual processing of information" (p. 141).

The rehearsal strategies include three elements where the participants: (1) read the problem and associate it with the relevant mathematics topic/content, (2) evoke prior knowledge relevant to the problem, and (3) highlight or underline important words or phrases. The elaboration strategies consist of three elements where the participants: (1) break down the problem into parts, (2) refer to previously seen problems, and (3) discuss the problem to clarify goals. The organisational strategies include four elements where the participants: (1) gather important information or facts from the problem that can help to solve the problem, (2) discuss and confirm the goals to achieve, (3) evaluate (a variety of) strategies, and (4) implement one chosen strategy.If a learner employs any of the elements of a component, and influences others in the group, the group is observed to engage with that particular component.

SRL and discourse: SRL strategies are found to be one of the factors in enhancing students' academic achievement (Zimmerman & Martinez-Pons, 1986). Wang, Haertel and Wahlberg (1990) show that high achievement learners engaged more on self-regulative activities, such as orientation, planning, monitoring, re-adjustment of strategies, evaluation and reflection. Apart from SRL, mathematical discourse is also vital in the success of mathematical learning. As Sfard (2001) notes, "putting communication in the heart of mathematics education is likely to change not only the way we teach but also the way we think about learning and about what is being learned" (p. 13).Unfortunately, literature associating SRL and mathematical discourse together in mathematical learning is currently limited. Based on this, the study will focus on students' engagement with mathematical tasks in the light ofmathematical discourse and SRL strategies.

The study

This study employed qualitative methods with the observation approach using video-recording as the primary tool for data collection. Video data allowed the researcher, like Pirie (1996), "to re-visit the aspect of the classroom" through which "greater leisure to reflect on classroom events" was gained (Pirie, 1996, n.p). Griffee (2005) noted, video-recording provides an opportunity to reveal things that might go unnoticed.

The study lasted for six months and involved a group of four Year 9 students aged between fourteen and fifteen years old in one secondary school in the East of England. Video recordings focused on the group engaging in mathematical tasks (20 – 25 minutes towards the end of the one hour lesson). In addition, observational notes were kept and students' written work was taken into account to complement the video data for a more complete record of the actual situation.

A sequence of seven interacting, non-linear phases of Powell *et al*. (2003) model was used to analyse the video data. At the early phases, the process of viewing, listening, and describing the video data were carried out. During these processes, vignettes or episodes that were critical and significant to the study were recorded. This was followed by transcribing the critical events or episodes whereby recordings of participants' utterances and actions were fully transcribed in order to capture both what was said and what was done. This was followed by the *coding* phase whereby all critical episodes were analysed employing the mathematical communication framework (Figure 1), a combination of discourse analysis (Sfard & Kieran, 2001) and Pintrich's (1999) SRL model.

The episodes were analysed using discourse analysis to capture the ways in which students interacted with each other. The focal analysis focused on the coherence of the utterances involving the tripartite foci: *pronounced focus, attended focus, and intended focus*. This was followed by preoccupational analysis employing the interactivity flowchart. It focused on how students communicate between different channels of communication and different level of talks (Kieran, 2001). The episodes were also analysed in-depth to scrutinise students' engagement with the SRL strategies, particularlythe cognitive learning strategies while working on the mathematical task.

For the purpose of this paper, the triangle problem (Figure 2) was selected as an exemplification of students' interactions and their engagement with the SRL strategies. This exercise was set to the students as part of a lesson on triangles and parallel lines. The students were given the diagram in Figure 2 and asked to find the angles *p, q, m* and *n*.The content of the lesson was on the properties of triangles and parallel lines including: (1) vertically opposite angles are equal, (2) alternate angles are equal, (3) corresponding angles are equal, and (4) supplementary angles add up to 180˚. In addition, previously, the students were taught about angles in polygon, and lines and angles.

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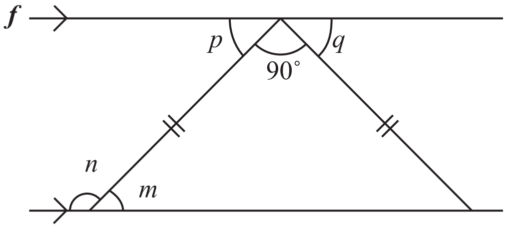

Figure 2 The triangle problem

The following conversation was recorded (time: 00:17:40 – 00:22:35):

[1] Megan: *n* and *m* are 90.
[2] Kathy: Are they?...no... oh yeah. (pause for a moment) Hah...they look like 90.
[3] Megan: So is *p*
[4] Kathy & Anne: No, they are 45 (referring to *p* and *q*).
[5] Kathy: Because they are same length.
[6] Megan: That is what does it... they will be the same, *m* and *p* (pointing at *m* and *p*).
[7] Kathy: Oh..yeah..
[8] Anne: Yes, that is not 90 (referring to *n* and *m*)...that means *m* is 45...
[9] Kathy: Let start again.
[10]Anne: *m* will be 45.
[11] Kathy: Do *n* first.
[12] Anne: Do this first (pointing at *p* and *q*). That's 45 each.
[13] Kathy: So *p* and *q* are 45.
[14] Megan: They will be equal, right?
[15] Kathy: Because they got them though (pointing at the two small lines which mean equal length).
[16] Anne: *n* is definitely 90.
[17] Kathy: No, it's not.
[18] Megan & Anne: Yes, it is.
[19] Kathy: No, it is not a zig-zag (referring to the alternate angles between *n* and 90)
[20] Megan: Yeah...it has to be that one (showing the top parallel line with the *q*-angle). It has to be like that (as though she is drawing the z). So *n* is not 90.
[21] Anne: Alright... so *p, q* are definitely 45... Then that would be 45 (pointing at *m*) and that would be 45 (drawing an interior angle on the left of the triangle)... because these are the same length.
[22] Kathy: Yeah...
[23] Anne: So *m* is definitely 45...and then so is *p*... and so is *q*... and then n is 145...
[24] Kathy: No.
[25] Megan: No. 135.
[26] Anne: Yeah.

Discourse analysis and cognitive learning strategies

The participants were interacting in an effective and productive mathematical discourse in two different segments of the episode. These segments involved utterances from [5] to [8] and from [19] to [23]. The flow of the participants' tripartite foci has been charted in Table 1. This is followed by the interactivity flowchart in Figure 3 that corresponds to the participants' interactions. At the same time, the participants' engagement with the cognitive learning strategies which included rehearsal strategies, elaboration strategies, and organisational strategies would be discussed.

Looking at Table 1, the focal analysis shows that in these two segments two different 'pronounced focuses' were found. In the first segment involving exchanges from [5] to [8] the 'pronounced focus' of the participants was centred on the sides with equal length. Thus, the 'pronounced focus' was the 'equal length' concept. The second segment which involved exchanges from [19] to [23] indicated that the participants were focusing on the parallel lines which had the connection to the unknown angle, *n*. In this segment, the participants' 'pronounced focus' was found to be the 'alternate angles' concept.

Table 1 The participants' tripartite foci of the Triangle problem

Megan			Kathy		
Pronounced Focus	**Attended Focus**	**Intended Focus**	**Pronounced Focus**	**Attended Focus**	**Intended Focus**
[6a] That is what does it	Diagram	Solution for *m* and *p* angles	[5] Because they are the same length	Diagram	Solution for *p* and *q* angles
[6b] they will be the same	Diagram	Solution for *m* and *p* angles			
[20a] Yeah… it has to be that one	Diagram	Solution for *n*-angle	[19] No, it is not a zig-zag	Diagram	Solution for *n*-angle
[20b] It has to like that	Diagram	Solution for *n*-angle			

	Anne	
Pronounced Focus	**Attended Focus**	**Intended Focus**
[8] that means *m* is 45	Diagram	Solution for *m*-angle
[21] Alright	Diagram	Solution for *n*-angle
[23] … andthen *n* is 145	Diagram	Solution for *n*-angle

The 'pronounced focus' or the 'concepts' was observed to arise from the participants' engagement with the rehearsal strategies, as they evoked their prior knowledge that was relevant to the task, in this case the prior knowledge of lines, parallel lines and angles. In these segments, Kathy was observed to be the dominant figure to guide others to focus on the relevant concepts that conformed to the unknown angles, *m, n, p,* and *q*. For example, in the first segment Kathy proposed the concept of 'equal length' not only to reject Megan's solutions but also to support her solution for *p* and *q* angles stating, *"Because they are the same length"* [5]. This could be seen through the interactivity flowchart (Figure 3) in which Kathy's utterance was a pro-action and re-action utterance which implied that she was not only reacting to Megan's statement [3] but also proposing a mathematical justification to the solution she proposed.

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In the second segment, Kathy's pro-action and re-action utterance (as shown in Figure 3 the interactivity flowchart) was observed not only objecting to the solution for *n* proposed by Anne [16] but also justifying that the *n*-angle did not make a zig-zag shape with the *90*-angle as she noted that, "*No, it is not a zig-zag*" [19]. Like the first segment, Kathy was using her prior knowledge of properties of parallel lines and angles which suggested that she was engaged with the rehearsal strategies. Kathy's justification in these two segments was observed to be a stimulus for other participants to focus on the 'pronounced focus' as

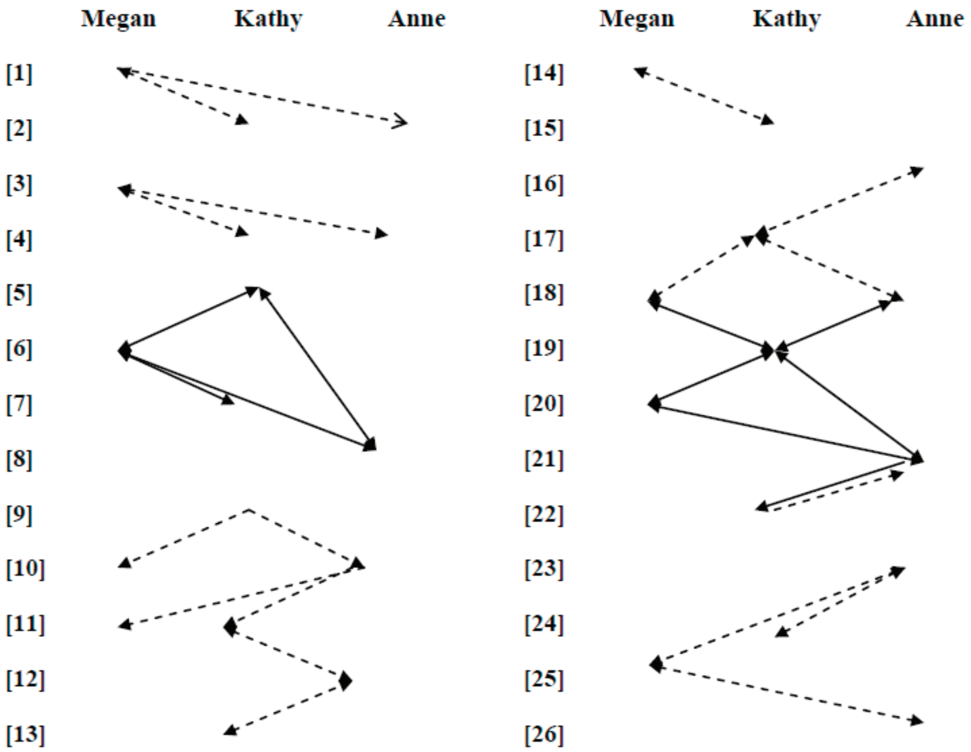

Figure 3 The interactivity flowchart of the Triangle problem

it was associated to the unknown angles. Thus, this helped others in the group to recall their prior knowledge of lines, parallel lines, and angles which enabled them to engage with the rehearsal strategies.

Reacting to Kathy's statement [5], Megan not only agreedwith it but also insisted that her solutions of *m* and *p* angles were similar based on the concept stating that, "*That is what does it… they will be the same, m and p*" ([6a] and [6b]). On the other hand, Anne used the 'equal length' concept to determine the *m*-angle as she noted that, "*...that means m is 45*..." [8]. The preoccupational analysis shows that these interpersonal exchanges, [6] and [8], were of mathematical communication and suggested that Megan and Anne were using their prior knowledge of lines and angles in agreeing with Kathy's argument on the 'equal length' concept. In doing so, Megan and Anne were observed to engage with the rehearsal strategies and, more importantly, they shared the same 'pronounced foci'.

In the second segment, the participants were focusing on the parallel lines which had the connection to the unknown angle, *n*. Kathy's idea was observed to guide the other participants to direct their attention to the angles associatedwith the parallel lines. More importantly, the others in the group were observed to engage with the rehearsal strategies by evoking their prior knowledge of alternate angles which became their 'pronounced foci'. Megan responded positively and demonstrated the angles that were alternate and associatedwith the parallel lines noting that, "*Yeah… it has to be that one*" [20a] and "*It has to be like that*" [20b]. In a similar tone, Anne used the 'alternate angles' concept to agree with Megan [21] and to confirm the value of *n* [23]. Like the first segment, these exchanges (from [19] to [21]) were observed to be mathematically connected through the use of a same 'pronounced focus'.

The second column of the 'attended focus' tells us that the participants shared their focus of attention as they were observed using the diagram of the triangle as a primary source of information. Thus, this suggests that the participants were engaged with the organisational strategies: gather information or facts from the problem that could help to solve the problem. For instance, the participants managed to identify the sides with equal length through the marks on the sides of the triangle. Again, in the later stage the participants used the diagram to show the connection between the parallel lines and the unknown angles. Using the information from the diagram, the participants justify the 'equal length' concept and the 'alternate angles' concept which in the end became the participants' 'pronounced foci'.

The focal analysis in Table 1 also shows that the participants' 'intended focus' was to find the values of the unknown angles, *m, n, p*, and *q*. In order to achieve the goal of the task, the participants discussed the task, proposing and reacting to others using mathematical justification to clarify their ideas. Thus, this suggests that the participants were engaged with the elaboration strategies: discuss the problem to clarify goals in finding the unknown angles. The preoccupational analysis (Figure 3 interactivity flowchart) illustrates that the exchanges from [5] to [8] and from [19] to [21] were interpersonal utterances of object-level communication which suggested that in these segments the participants were interacting mathematically with others in the group. More importantly, the participants' interactions formed a closed triangular pattern with no gaps in between which implied at that moment the participants were interacting not only mathematically but also developing a meaningful and productive discourse.

Apart from the exchanges (from [5] to [8] and from [19] to [21]) discussed earlier, the preoccupational analysis throughthe interactivity flowchart showed that the participants' interactions formed a non-patterned formation. The non-patterned formation demonstrated that the participants were interacting through pro-action or re-action utterances which meant that the participants were either proposing or reacting to other participants' utterances but not both at the same time. Thus, the interactions were observed to be loose with a lot of gaps in between which suggestedthat although the participants were discussing a mathematical problem no meaningful mathematical discourse took place.

For instance, Megan begian with a pro-action utterance stating that, "*n and m are 90*" [1] which Kathy reacted in doubt stating that, "*Are they?...no... oh yeah. Hah...they look like 90*" [2]. Megan again suggested that, "*So is p*" [3] which was rejected by Kathy and Anne stating that, "*No, they are 45*" [4]. Looking at these four exchanges, it was discovered that the interactions were more towards 'one proposes one rejects' with no indication of meaningful mathematical learning taking place. During these exchanges, the participants were observed not to engage with any elements of the cognitive learning strategies. For instance, Megan was observed to propose the values of the unknown angles just by inferring from the diagram with no stated mathematical justification. Unlike Megan, Kathy and Anne were observed to infer from the diagram, and also provided the correct values for *p* and*q*. This could inferred that there was a missing link between the participants' exchanges that restricted the development of a productive discourse.

The existence of non-patterned discourse was also discovered in the middle and at the end of the discourse involving utterances from [9] to [15] and from [22] to [26]. Sincethey had discussed the possible outcomes for *m, p*, and *q* angles earlier on, exchanges from [9] to [15] showedthe participants' interactions were more on restructuring the process of which unknown angle should they worked on first. For example, Kathy suggested, "*Do n first*" [11]. On the other hand, Anne proposed that they worked on *p* and *q* angles first [12] which Kathy agreed to in [13]. The exchanges also involved repetition of mathematical facts that had been discussed, as shown in utterances [14] and [15]. In order to confirm the solution for *p* and *q* angles, Megan inquired, "*They will be equal, right?*" [14] and Kathy responded confirming that, "*Because they got them though*" [15]. These repetition exchanges were observed to slow down the development of a productive discourse and at the same time no cognitive learning strategies were involved. Thus, at this point the participants were not regulating their learning as a group.

Like exchanges from [9] to [15], exchanges from [22] to [26] demonstrated the participants' interactions in confirming and finalising the solutions for the unknown angles. Again, this was observed to be a repetition of what had been discussed as the solutionsobtained and therefore the exchanges would have little effect in determining the productiveness of the discourse. Besides that, the participants were not engaged with cognitive learning strategies as they had already made their decision on the values of the unknown angles, *m, n, p*, and *q* earlier.

Discussion

In the *Triangle* problem (Figure 2), it was discovered that the participants were observed to interact effectively and productively in two different segments of the episode involving the 'equal length' concept and the 'alternate angles' concept in order to find the values for unknown angles. This can be observed through the participants' exchanges from [5] to [8] and from [19] to [23]. The emergence of these key concepts had provided a positive impact towards generating a mathematical discourse among the participants. The focal analysis showed that the participants' interactions were centred on these concepts and most importantly the participants shared the same tripartite foci. These concepts became the 'pronounced foci' of the discourse which saw a coherence in speech among the participants. The 'pronounced focus' was observed as the participants were engaged with the rehearsal strategies at group level when they evoked their prior knowledge of line, parallel lines, and anglesrelevant to the task.

Apart from that, the participants were also observed to have the same attention and intention in developing a more meaningful mathematical discourse. The participants' 'attended focus' was found to be the diagram of the triangle (Figure 2) where they gathered important information, such as equal length sides and property of parallel lines (alternate angles). Apparently, this suggested that the participants were engaged with the organisational strategies. The focal analysis also showed that the participants' 'intended focus' was to find the unknown angles, *m, n, p*, and *q*. Throughout the task, the participants were discussing and justifying the values for the unknown angles which suggested their intention and at this point they were observed to engage with the elaboration strategies.

Since the participants were sharing the same tripartite foci in these two segments, theywere involved in an effective discourse towards the goal of the task. This was supported by the preoccupational analysis. Looking at the interactivity flowchart (Figure 3) the participants' interactions were of pro-action and re-action utterances involving the use of these concepts. The emergence of these concepts was observed to influence the discussion as the participants were able not only to interact mathematically with others in the group but also to develop a meaningful and productive discourse.

However, in parts of the discussion in the *Triangle* problem, excluding the two segments discussed earlier, the preoccupational analysis showed that the participants' interactions formed a non-patterned shape. This was indicated as non-productive discourse. During this segment, the participants were observed not to share the same common tripartite foci and they were not engaged with any elements of the cognitive learning strategies.

Conclusion

This study investigated students' interactions along with their engagement with the SRL strategies while working on mathematical tasks. Two different approaches were implemented in the investigation. The study's approach employed the mathematical communication framework, an integration of Sfard and Kieran's (2001) discourse analysis framework and Pintrich's (1999) SRL model in order to examine student's mathematical discourse and their engagement with the cognitive learning strategies. The findings suggested that during the participants' engagement with the SRL strategies, it was observed that the emergence of two key mathematical concepts that were not only significant to the problem but also crucial to the development of an effective and productive discourse. The participants' interactions involving these concepts could be identified clearly through the interactivity flowchart as the utterances formed a pattern of closed triangular and rectangular shapes which suggested a productive discourse. Besides that, applying these concepts encouraged the participants to focus and talk on a similar mathematical object. Thus, this produced an effective discourse.

To conclude, the study showed that students were fully involved in an effective and productive discourse and were engaged with the components of the cognitive learning strategies.

References

Artzt, A. F., &Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and Instruction,* 9(2)*,* 137 – 175.

Griffee, D. T. (2005). Research tips: Classroom observation data collection, part II. *Journal of Development Education*, 29(2)*,* 36 – 37.

Kieran, C. (2001). The mathematical discourse of 13-year-old partnered problem solving and its relation to the mathematics that emerges. *Educational Studies in Mathematics,* 46(1-3), 187 – 228.

ISSN 1985-7918

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Pintrich, P. R. (1999). The role of motivation in promoting and sustaining self-regulated learning. *International Journal of Educational Research,* 31(6), 459 – 470.

Pirie, S. E. B. (1996). Classroom video-recording: when, why and how does it offer valuable data source for qualitative research?*Proceedings of the 16th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*.Panama City, Panama.

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data.*Journal of Mathematical Behavior,* 22(4), 405 – 435.

Ryve, A. (2004). Can collaborative concept mapping create mathematically productive discourses? *Educational Studies in Mathematics,* 56(2/3), 157 – 177.

Ryve, A. (2006). Making explicit the analysis of students' mathematical discourses – revisiting a newly developed methodological framework. *Educational Studies in Mathematics,* 62(2), 191 – 209.

Sfard, A. (2001). There is more to discourse than meets the ears: looking at thinking as communication to learn more about mathematical learning.*Educational Studies in Mathematics*, 46(1-3), 13 – 57.

Sfard, A., & Kieran, C. (2001). Cognition as communication: rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity,* 8(1), 42 – 76.

Wang, M. C., Haertel, G. D., and Wahlberg, H. J. (1990). What influences learning? A content analysis of review literature.*The Journal of Educational Research*, 84(1), 30 – 44.

Zimmerman, B. J., and Martinez-Pons, M. (1986). Development of a structured interview for assessing students' use of self-regulated learning strategies.*American Educational Research Journal*, 23(4), 614 – 628.