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Real Scalar Spin-0 Field in Five Dimensions

Medan Spin-0 Skalar Nyata dalam Lima Matra

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Abstract

We generalized the Kaluza-Klein theory of massive real scalar spin-0 field in five dimensions with self-interaction term Φ^4. Fifth coordinate undergoes compactification and periodical. In potential term, we derived the self-interaction of different modes of field with strength 2λ and the complete Lagrangian. We determined the mass of field related to their modes. We also discussed the dimensional reduction D = 5 → D = 4 of real scalar field, where in four dimensional spacetime we just observed the five dimensional zero mode of real scalar field.

Keywords Kaluza-Klein theory, real scalar spin-0 field, five dimensions

Abstrak

Kita mengitlakkan teori Kaluza-Klein ke atas medan spin-0 skalar nyata dalam lima matra dengan ungkapan swa-interaksi Φ^4. Koordinat kelima mengalami kemampatan dan bersifat berkala. Dalam ungkapan keupayaan, kita menerbitkan swa-interaksi berbeza mod medan dengan kekuatan 2λ dan pada Lagrangian lengkap. Kita menentukan jisim medan yang menghubung kepada bilangan mod medan. Kita juga membincangkan penurunan bermatra D = 5 → D = 4 medan scalar nyata, yang mana dalam ruangmasa empat matra kita hanya mencerap medan scalar nyata lima matra pada mod sifar.

Kata kunci Teori Kaluza-Klein, medan spin-0 skalar nyata, lima matra

Introduction

Extra dimensional spacetime is one of the mathematical ingredients for Theory of Everything which unifies all four forces, besides supersymmetry. The earliest work in this area was started by Theodor F. E. Kaluza (Kaluza, 1921), who proposed that electromagnetism can be unified with gravitation in five dimensions. This is followed by Oscar Benjamin Klein (Klein, 1926 &1927) who treated the extra dimension seriously but he assumed the fifth dimension to have a circular topology so that the coordinate y is periodic, $0 \leq ny \leq 2\pi$, where n is the inverse radius of the circle S_1

Recently, string theory naturally requires ten dimensions to live, where the Kaluza-Klein framework played a role. Obviously, we are experiencing only four dimensions, therefore, if extra dimensions exist, it is supposed to have a mechanism making them invisible to us. The compactification is the most convenient to realise the hidden extra dimensions, i.e. where the extra dimensions are compactified with small radii that we as human beings cannot see it even with current accelerator facilities.

There are a lot of researches on Kaluza-Klein theory such as Overduin and Wess, (1997) who studied the Kaluza-Klein theory in connection to the general relativity and the cosmological and astrophysical implications of extra dimensions. Dolan and Szabo, (2009) studied the dimensional reduction of gauge theories in a way which naturally incorporates the topology of gauge fields on coset G/H and there are already excellent review articles regarding the Kaluza-Klein theory (Bailin & Love, 1987 and Duff, 1994).

In this paper, we study a Kaluza-Klein theory with a real scalar spin-0 in five dimensions. Our fifth dimension is independent, compactified and periodical coordinate. We generalize the field part by part in this paper, starting with potential term and followed by kinetic term in five dimensions. Then, we discuss the Kaluza-Klein reduction as well, i.e. the dimensional reduction from five dimensions into four dimensional spacetime, with an extra dimension hidden.

Kaluza-Klein Theory

In Kaluza-Klein theory we consider the extra dimension (denoted as *y*) for the real scalar field to be independent from four dimensional and it undergoes compactification. The extra dimension is represented as a circle of a small radius for its topology. Then our five dimension space-time will have the $M_4 \times S_1$ geometry, where M_4 is the four-dimensional Minkowski space-time and S_1 is the circle of extra dimension. The extra dimension is periodic, $0 < n\,y < 2\pi$. Because of the small radius, in our real life we will only perceive four dimensional space-time, and we will not observe the extra dimension.

We treat the periodicity of the extra dimension using a Fourier expansion with an infinite expansion of fields in four dimensions. The first term corresponds to the reduction of Kaluza-Klein theory. First we establish our convention, where four Minkowski space-time indices are represented by $\mu, \nu = 0, 1, 2, 3$ and for five dimensional spacetime we denote them as $M,\ N = 0, 1\ 2, 3, 4$ and the five-dimensional coordinates are (x^μ, y), where x^μ and y are the spacetime coordinates and fifth dimension coordinate respectively. The real scalar field in five-dimension reads, $\Phi(x^\mu, y)$. But the extra-dimension would be periodic at $y \in [0, 2\pi]$, so we represent the field in terms of a Fourier expansion,

$$\Phi\left(x^\mu, y\right) = \sum_{n=0}^{\infty} \phi_n\left(x^\mu\right) e^{\frac{iny}{L}} \tag{1}$$

where $\phi\left(x^\mu\right)$ is the scalar field in four dimensional spacetime and *L* is the small radius of compactified extra dimension, which is taken to be of Planck radius (10^{-33}cm, it is very much smaller than the 10^{-16} cm achievable by current particle accelerators).

Φ^4 Theory

In this section we attempt to investigate the compactification of fifth dimension for an interacting real scalar field whose Lagrangian includes the quartic term Φ^4. Let us start with

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the following massive real scalar spin-0 field Lagrangian in five space-time dimensions.

$$L_{(5)} = \frac{1}{2}\partial_M \Phi^* \partial^M \Phi - V\left(\Phi^*\Phi\right) \tag{2}$$

where the five dimension spacetime derivative is denoted by $\partial_M = \partial_\mu + \partial_y$ and the potential in five space-time dimensions reads:

$$V\left(\Phi^*\Phi\right) = \frac{1}{2}\mu^2 \Phi^*\Phi + \frac{1}{4}\lambda\left(\Phi^*\Phi\right)^2 \tag{3}$$

Potential Term in Five Dimensions

The Lagrangian given by equation (2) is invariant under the spatial-inversion (i.e. $\Phi \rightarrow -\Phi$) with the features of the tachyonic condensation (i.e. condensate for an imaginary mass with $\mu^2 < 0$). We consider the term for potential term after substitution of equation (1) to (3) and it given,

$$V\left(\Phi^*\Phi\right) = \frac{1}{2}\mu^2\left(\sum_{n=0}^{\infty}\phi_n^*\left(x^\mu\right)e^{-\frac{iny}{L}}\right)\left(\sum_{n=0}^{\infty}\phi_n^*\left(x^\mu\right)e^{\frac{iny}{L}}\right) + \frac{1}{4}\lambda\left(\left(\sum_{n=0}^{\infty}\phi_n^*\left(x^\mu\right)e^{-\frac{iny}{L}}\right)\left(\sum_{n=0}^{\infty}\phi_n^*\left(x^\mu\right)e^{\frac{iny}{L}}\right)\right)^2$$

(4)

To solve that, we consider the real scalar field in four dimensions spacetime is an orthogonalized field, so we treat those fields must satisfy following conditions;

$$\begin{aligned} \phi_n^*\phi_m &= 0 \qquad if \qquad n \neq m \\ \phi_n^*\phi_m &= \left|\phi_n\right|^2 \qquad if \qquad n = m \end{aligned} \tag{5}$$

From the conditions, we simplified the potential term by,

$$V\left(\Phi^*\Phi\right) = \frac{1}{2}\mu^2\left(\sum_{n=0}^{\infty}\left|\phi_n\left(x^\mu\right)\right|^2\right) + \frac{1}{4}\lambda\left(\sum_{n=0}^{\infty}\left|\phi_n\left(x^\mu\right)\right|^4 + 2\sum_{n\neq m}^{\infty}\left|\phi_m\left(x^\mu\right)\right|^2\left|\phi_n\left(x^\mu\right)\right|^2\right) \tag{6}$$

We found that fifth dimensional factor are eliminated from the potential, and the expansions provides three sums in the potential, which have different interpretations. The first sum of the equation shows the mass terms of expansion modes of the four dimensional spacetime field which has the same mass μ . The second sum is the self-interaction of the field where the strength of self-interaction is λ, while the third sum implies that there exists the self-interaction between different modes of fields with strength of self-interaction of 2λ .

Kinetic Term in Five Dimensions

Next we will consider the kinetic term which has the following form after considering the Fourier expansion of fields,

$$\frac{1}{2}\partial_M\Phi^*\partial^M\Phi=\frac{1}{2}\partial_M\left(\sum_{n=0}^{\infty}\phi_n^*\left(x^\mu\right)e^{-\frac{iny}{L}}\right)\partial^M\left(\sum_{n=0}^{\infty}\phi_n\left(x^\mu\right)e^{\frac{iny}{L}}\right) \quad (7)$$

With a similar condition of orthogonalized fields, our kinetic term becomes,

$$\frac{1}{2}\partial_M\Phi^*\partial^M\Phi=\sum_{n=0}^{\infty}\left(\frac{1}{2}\partial_\mu\phi_n-\frac{1}{2}\frac{n^2}{L^2}\left|\phi_n\right|^2\right) \quad (8)$$

The compactification of fifth dimension in the kinetic term has contributed another mass term for our four dimensional spacetime real scalar field $\phi_n(x^\mu)$.

Thus the complete Lagrangian gives,

$$L_{(5)}=\sum_{n=0}^{\infty}\frac{1}{2}\partial_\mu\phi_n^*\partial^\mu\phi_n-\sum_{n=0}^{\infty}\left|\phi_n\left(x^\mu\right)\right|^2\left(\frac{1}{2}\frac{n^2}{L^2}+\frac{1}{2}\mu^2\right)-\frac{1}{4}\lambda\sum_{n=0}^{\infty}\left|\phi_n\left(x^\mu\right)\right|^4-\frac{1}{2}\lambda\sum_{n\neq m}^{\infty}\left|\phi_m\left(x^\mu\right)\right|^2\left|\phi_n\left(x^\mu\right)\right|^2$$

(9)

This means that $\phi_n(x^\mu)$ are four dimensional scalar fields with masses,

$$M_\phi^2=\frac{n^2}{L^2}+\mu^2 \quad (10)$$

This again shows that the mass of each field is different according to their mode, but they have the same self-interaction with itself and with different modes.

Kaluza-Klein Reduction

When we consider the small radius limit $L\to 0$, it will give the consequence that $\frac{n^2}{L^2}\to\infty$. This means that the fields with modes $n\neq 0$ give an infinite power of massive real scalars with mass $\frac{n}{L}$ and it could be discarded. In fact, we refer the limit where the only zero mode is considered as dimensional reduction. The complete Lagrangian will become,

$$L_{(5)}\to L_{(4)}=\frac{1}{2}\partial_\mu\phi_0^*\partial^\mu\phi_0-\frac{1}{2}\mu^2\left|\phi_0\right|^2-\frac{1}{4}\lambda\left|\phi_0\right|^4 \quad (11)$$

The mass of scalar field become $M_\phi=\mu$ as well, and the results imply that in four dimensional spacetime scalar field is just the zero mode field of five dimensional real scalar field with the infinite mode fields of Fourier expansion being discarded and decoupled from low-energy physics.

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Conclusions

In this work, we have generalized the massive real scalar spin-0 field with renormalizable potential Φ^4 term to five dimensions using the Kaluza-Klein theory. The compactification and independence of the fifth coordinate was included in the field. The work resulted in that self interaction of different modes given twice as strong as the self-interaction itself and we provide the new representation of mass in five dimensional spacetime.

We provided next the Kaluza-Klein reduction to give our real scalar field a physical sense in four-dimensional spacetime and we found the zero mode reproduces the normal 4-dimensional scalar with the infinite modes discarded. The results coincided with the fact that small radius circle L approaches the Planck radius of 10^{-33}cm, where it is very much smaller than the 10^{-16} cm achievable by current particle accelerators.

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