

ENHANCING SINGLE-DIGIT ADDITION SKILLS IN REMEDIAL PUPILS USING LAURNAL'S FRAME: A CASE STUDY

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ABSTRACT

This research introduces "Laurnal's frame," a novel visual tool designed to improve remedial pupils' understanding of single-digit addition. Rooted in Bruner's enactive, iconic, and symbolic modes of representation, Laurnal's frame comprises a rectangular frame divided into ten sections representing numbers 1 through 10, with blocks for hands-on, enactive learning. The study involved one remedial teacher and four remedial pupils, aged 8, who used Laurnal's frame over a period of four weeks for addition instruction. The researchers employed a qualitative case analysis study design, and a purposive sampling method was used to obtain a sample of four remedial pupils and one remedial teacher. Data were gathered in in-depth interview, direct observations, and analysis of documents such as worksheets and journal entries were performed to gain a comprehensive picture of remedial pupils making sense of number patterns. Results indicated significant gains in addition skills, with pupils showing enhanced comprehension of number bonds through both iconic visualization and symbolic understanding. The structured design of Laurnal's frame promoted interactive learning, increased enthusiasm, and fostered a positive attitude towards mathematics. The findings suggest that integrating Laurnal's frame into early math curricula can potential useful bolster foundational arithmetic skills and create a supportive learning environment.

Keywords: Laurnal's frame, single-digit addition, remedial education, visual learning tool, Bruner's modes of representation, early arithmetic skills

INTRODUCTION

Mathematics is essential for academic learning because it enhances logical thinking and problem-solving abilities (Ahdhianto et al., 2020). It encompasses a range of transferable skills that enable pupils to collaborate with peers, discover, analyze, synthesize, and apply knowledge in new situations (Darling-Hammond et al., 2020). Addition, described as "the joining of two sets" (Fuson, 1992), is fundamental for mastering other mathematical skills. Studies show that early proficiency in addition predicts later mathematical achievement in children (Clerkin & Gilligan, 2018; Raghubar & Barnes, 2017). Furthermore, pupils who received early numeracy intervention demonstrated improved math performance in second grade (Bryant et al., 2021).

Understanding single-digit addition is a foundational skill in early mathematics education, crucial for developing more complex arithmetic and problem-solving abilities (Sunde et al., 2023). Carpenter and Moser (1984) identified three levels of addition strategies: direct modelling, counting strategies, and number fact strategies. Direct modelling involves

using fingers or objects to count all items, while counting strategies progress from counting all items starting with the first addend to counting on from the larger addend, which precedes recalling number facts (Sarama & Clements, 2009). Number fact strategies involve either direct retrieval or derived fact strategies.

Jerome Bruner's theory of cognitive development identifies three modes of representation enactive, iconic, and symbolic that are essential in the learning process (Ozdem-Yilmaz & Bilican, 2020). The enactive mode involves learning through direct manipulation and physical activity, the iconic mode through visual images and diagrams, and the symbolic mode through abstract symbols and language. Haylock & Manning (2019) emphasized the importance of making various types of connections such as linking language with symbols, pictures with language, or real-life experiences with symbols—to enhance learners' understanding. These modes underscore the importance of using varied and interactive teaching tools to cater to different learning styles and stages of cognitive development in young children.

This study explores the effectiveness of the Laurnal' frame in teaching single-digit addition to remedial pupils aged 6. By integrating Bruner's enactive, iconic, and symbolic modes, the Laurnal' frame aims to enhance understanding and retention of addition concepts. Over a four-week period, pupil interacted with the Laurnal' frame, and their progress was measured through pre- and post-tests, as well as observational data on engagement and interaction. This introduction sets the stage for examining how the Laurnal' frame can improve mathematical proficiency and foster a positive learning experience.

LITERATURE REVIEW

The early years of math education are crucial for setting the foundation for future learning (Clerkin & Gilligan, 2018). In Grade One, mastering single-digit addition is a key milestone (Sunde et al., 2023). Research from 2020 to 2022 has explored various strategies to enhance the effectiveness of teaching single-digit addition. This literature review examines the different approaches and their impact on student learning outcomes. The Concrete-Representational-Abstract (CRA) approach involves teaching math concepts through concrete materials, then representational drawings, and finally abstract symbols. Studies such as those by Jones and Tiller (2020) and Bartolini & Martignone (2020) have shown that the CRA approach significantly improves pupils' understanding and retention of single-digit addition. These studies highlight the importance of a structured progression from tangible objects to abstract concepts. Manipulatives like blocks and number lines are widely used to teach addition. Research by Kwon & Capraro (2021) found that using manipulatives helped pupils visualize addition problems, leading to better comprehension and accuracy. Learning aids, as discussed by Fatwana et al., (2023), provide additional support, particularly for visual learners, and enhance engagement.

Traditional tools like the ten frames have been widely used to help pupils visualize and understand addition concepts (Laux, 2022). A ten frame is a two-by-five rectangular frame where children can place blocks to visually represent numbers up to ten. Research has shown that ten frames help children develop number sense, understand part-whole relationships, and perform basic addition and subtraction (Fuson & Briars, 1990). However, while ten frames are effective, they primarily focus on numbers up to ten, and there is a need for tools that extend this understanding to a broader range of numbers. Here I argue that the "Laurnal' frame" is a new teaching tool designed to meet this need.

It is a rectangular frame divided into ten sections, each representing a number from 1 to 10, and is intended to facilitate hands-on, visual, and symbolic learning of single-digit addition (see Fig.1).

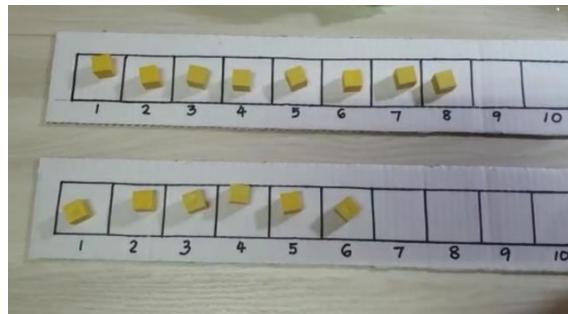


Figure 1: Laurnal' frame Bruner's Modes of Representation

Bruner (1966) proposed that cognitive development progresses through three stages of representation. The enactive mode involves learning through actions and physical manipulation of objects, making it a vital stage for young learners who benefit from hands-on activities. The iconic mode involves learning through images and visual aids, which help children make connections between concrete experiences and abstract concepts. The symbolic mode involves the use of language and symbols to represent ideas, which is crucial for more advanced mathematical thinking. Bruner's framework underscores the importance of using diverse and interactive teaching methods to support children's cognitive development.

Innovative visual learning tools have emerged to address the limitations of traditional methods. These tools aim to enhance engagement and provide a more comprehensive understanding of mathematical concepts. For instance, Sarama & Clements (2009) highlighted the importance of dynamic and interactive visual aids in improving mathematical comprehension. Tools that incorporate movement, visual representation, and symbolic thinking can bridge the gap between concrete and abstract understanding. Moreover, research indicates that visual and hands-on tools significantly enhance young learners' mathematical abilities. Fatwana et al., (2023) found that pupils who used visual aids and manipulatives developed a deeper understanding of mathematical concepts compared to those who relied solely on traditional methods. Moreover, Hinton et al., (2024) emphasized that engaging, hands-on activities can increase motivation and positive attitudes towards mathematics, leading to better learning outcomes. Hereby we argue that the Laurnal' frame is a novel teaching tool designed to incorporate Bruner's enactive, iconic, and symbolic modes of representation. It consists of a rectangular frame divided into nine sections, each representing a number from 1 to 9. This design allows children to physically manipulate blocks (enactive mode), visualize number relationships (iconic mode), and connect these experiences to numerical symbols (symbolic mode). By encompassing these modes, the Laurnal' frame aims to provide a holistic learning experience that caters to different stages of cognitive development.

In detail, the Laurnal' frame is a rectangular frame divided into ten equal sections, each representing a number from 1 to 10. Blocks are used to facilitate hands-on interaction. The frame's design aligns with Bruner's modes of representation: Enactive Mode: Pupils physically manipulate blocks to perform addition. Iconic Mode: Pupils visualize number relationships within the frame. Symbolic Mode: Pupils connect their hands-on and visual experiences to numerical symbols and addition sentences. The study was conducted over four weeks, with two 30-minute sessions per week dedicated to using the Laurnal' frame for teaching single-digit

addition. Each session included the following steps: Introduction (5 minutes): The concept of single-digit addition was introduced or reviewed, and instructions on using the Laurnal' frame were provided. Interactive Activity (20 minutes): Pupils worked individually or in pairs, using the Laurnal' frame to solve addition problems.

They physically manipulated blocks to represent and add numbers. Reflection and Discussion (5 minutes): Pupils shared their solutions and strategies, and the teacher facilitated a discussion to reinforce learning and address any misconceptions.

In concluding the paragraph, the Laurnal' frame represents an innovative approach to teaching single-digit addition, integrating Bruner's modes of representation to enhance learning. This literature review highlights the theoretical foundations and empirical evidence supporting the use of diverse and interactive teaching tools in early mathematics education. By providing a multi-modal learning experience, the Laurnal' frame has the potential to improve mathematical proficiency and foster a positive attitude towards learning among remedial pupils.

METHODOLOGY

This case study investigates the usage of the Laurnal' frame in teaching single-digit addition to remedial pupils. The methodology encompasses the design and implementation of the teaching tool, the selection and assessment of participants, and the data collection and analysis procedures. The study involved four remedial pupils aged 8 from a local elementary school. Participants were selected based on their age and current level of mathematical proficiency, ensuring a representation of remedial pupils. Parental consent was obtained for all participants, and ethical guidelines were followed throughout the study. Data were collected through observation, interview, and document analysis. The objectives of this study are to explore how "Laurnal's frame" remedial pupils in learning single digit addition within 20. Hence the research questions for this study are (1) How "Laurnal's frame" help learner in learning single digit addition has been implemented? The researchers collected the data from a single case study and then the data were interpreted in an integrated manner to answer the above research questions

Data for this study were gathered from multiple sources during a month-long teaching period. We analysed remedial pupils' worksheets and cross-referenced them with lesson observations and a 30-minute in-depth interview to ensure the trustworthiness of the findings (O'Leary, 2014). The key data used in this analysis were Noy's journal reflections, which she composed after her classes (Yin, 2018). Self-reports, such as these journals, provide valuable insights into the participant's perspective, independent of external realities (Yin, 2018).

The researcher conducted two lesson observations, during which anecdotal notes and photographs were taken. The primary goal of these observations was to explore Noy's teaching methods in helping remedial pupils learn single digit addition. Rich data sources were essential to capture the complex interplay of Noy's beliefs and values that influenced her teaching (Yin, 2018). After reviewing her observation notes, Noy documented her reflections in her journal. Additionally, she participated in a 30-minute interview to discuss her reactions to the intervention and her feelings about her teaching.

We utilized thematic analysis, as outlined by Clarke and Braun (2013), to analyse data from journal reflections, interview transcripts, and observational data. The analysis involved four distinct steps. Since all written entries were originally in Malay, the first step was to translate them into English. To ensure translation accuracy, the research team conducted cross-

checks. Categories were then developed based on themes that emerged from the journal entries and their relevance to the study.

In the second step, the audio recordings were transcribed to verify the accuracy of the transcripts. Data from Noy's reflective papers, interview transcripts, and lesson observation reports were analyzed, with additional categories generated as needed for the report. Throughout the analysis, we repeatedly reviewed and re-examined the coding principles to ensure that no new patterns emerged, thereby enhancing the trustworthiness and rigor of the results.

While this study provides valuable insights into the usage of the Laurnal' frame, certain limitations should be acknowledged: The sample size was relatively small and drawn from a single school, which may limit the generalizability of the findings. The study's duration was limited to four weeks, and longer-term effects were not assessed. The reliance on teacher and research assistant observations may introduce subjective bias. At last, the methodology outlined provides a comprehensive approach to evaluating the Laurnal' frame's impact on teaching single-digit addition. By integrating quantitative and qualitative data, the study aims to offer a thorough understanding of how this novel tool can enhance early mathematical education.

DATA FINDINGS

Background of the Research Participants

The four remedial pupils were each 8 years old. They communicated effectively with their friends and teachers using the Malay language. According to their previous remedial assessment results, they struggled with performing addition tasks.

The Teacher Participant (Noy – Pseudonym)

Noy is a remedial teacher at a national primary school in Malaysia. She holds a bachelor's degree in remedial education from a local Malaysian university. During a 2-hour face to face workshop, she was introduced to and given an explicit demonstration of how to use the Laurnal's frame as a teaching mechanism. Noy conducts three mathematics lessons (30 minutes each) with her year two remedial pupils every Monday, Wednesday, and Friday.

Two themes emerged in this present study: (1) Build conceptual understanding (2) Increased Engagement. This section provides a case profile of helping four remedial pupils to make sense of single digit addition. The case description clarifies how Noy help four remedial pupils to make sense single digit addition through Laurnal's frame at remedial class. The intended learning outcome for this lesson was being able to master single digit addition within 20.

Theme 1: Building Conceptual Understanding through Laurnal's frame

This section addresses the first research question: How "Laurnal's frame" help learner in learning single digit addition has been implemented? The findings revealed that Noy utilized Concrete-Representational-Abstract (CRA) approach, specifically through the use of "Laurnal's frame," to support remedial pupils in grasping single digit addition within 20. This approach allowed remedial pupils to build a deep conceptual understanding of these addition in an engaging and developmentally appropriate manner.

Visual Understanding: The iconic representation of numbers within the Laurnal' frame aided in visualizing number bonds. Pupils were able to see the relationship between numbers more clearly and used this understanding to solve addition problems more accurately. Before beginning the lessons, Noy prepared a Laurnal's frame along with blocks to use as teaching aids (see Fig. 2).



Figure 2 : Teaching aids: Laurnal's frame and blocks

In the interview, Noy explained: “First, I introduced the lesson’s objective—single-digit addition—to the pupils. I demonstrated how to add single-digit numbers and move the blocks according to the spaces shown on the Laurnal frame. For example, for $9 + 4$, I drew the Laurnal frame with an upper deck and a lower deck, each with ten sections. I placed 9 blocks on the upper deck and 4 blocks on the lower deck (see Fig. 3). Next, I pointed out that there was a space on the upper deck that needed to be filled. I moved the last block from the lower deck to the upper deck to fill this space. This allowed the pupils to clearly visualize that $9 + 4$ equals 13, as the Laurnal frame showed: the upper deck filled with 10 blocks and the lower deck with 3 blocks, with one block moved to the upper deck to complete the 10.”

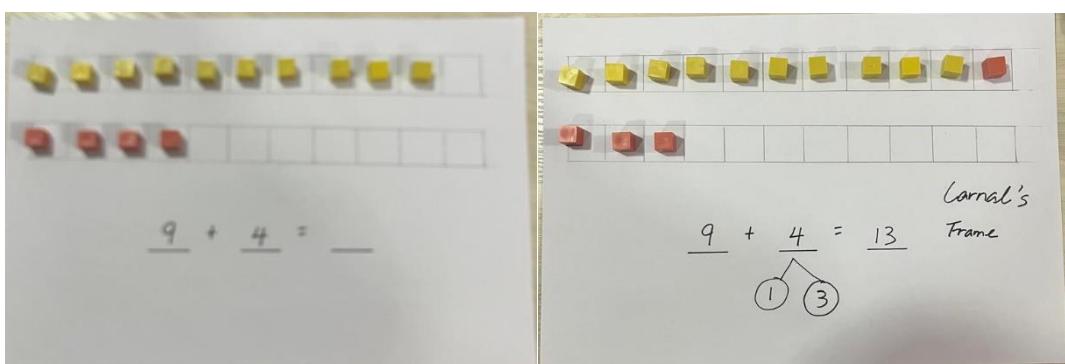


Figure 3 : Noy explains the concept of addition with regrouping using Laurnal's frame

Hands-On Learning: The enactive mode of learning, facilitated by manipulating blocks, helped pupils better understand addition concepts. Pupils frequently used blocks to experiment with different number combinations and verify their answers.

The pupils were encouraged to touch the blocks and the Laurnal's frame." She further elaborated: "They enjoy manipulate the blocks more than doing mathematics equations." Observation data showed that the pupils were excited to manipulate the blocks and move the blocks themselves. Noy demonstrated how to add the number on the Laurnal' frame and move the blocks in one-to-one correspondence. The pupils followed Noy's actions, speaking the number words aloud and moving the blocks accordingly (see Fig. 4).



Figure 4 : Remedial pupils manipulate the blocks and Laurnal's frame to gain the conceptual understanding of addition

In her reflective journals, Noy remarked: "The Laurnal's frame, a novel teaching tool, effectively trains young learners in addition when moving the block, emphasizes counting in one-to-one correspondence, and teaches them to recognize two rectangular frames divided into ten equal sections, each representing a number from 1 to 10. Additionally, the Laurnal's frame is used to teach single-digit addition with regrouping, providing a hands-on approach that helps pupils grasp the concept as many of my remedial pupils have difficulties in learning addition with regrouping, I think this is the foundation for addition."

Symbolic Representation: As sessions progressed, pupils increasingly connected their hands-on and visual experiences to symbolic representations. They began to write addition sentences more confidently and accurately.

Noy assessed the four remedial students on their ability to perform single-digit addition with regrouping within 20 in the final step of the lesson. Noy demonstrated the addition process again, emphasizing how to regroup and correctly identify the sum. After the lessons, Noy shared in her interview: "It was great that the students mastered single-digit addition with regrouping within 20 after the sessions." The results showed that the students could correctly solve addition problems and demonstrate regrouping by filling in the Laurnal's frame (see Fig. 5). Additionally, they were able to perform regrouping and complete various addition problems accurately (see Fig. 6). Moreover, the students were able to write the correct answers on their worksheets.



Figure 5 : Pupil A can correctly solve addition problems and demonstrate regrouping by filling in the Laurnal's frame

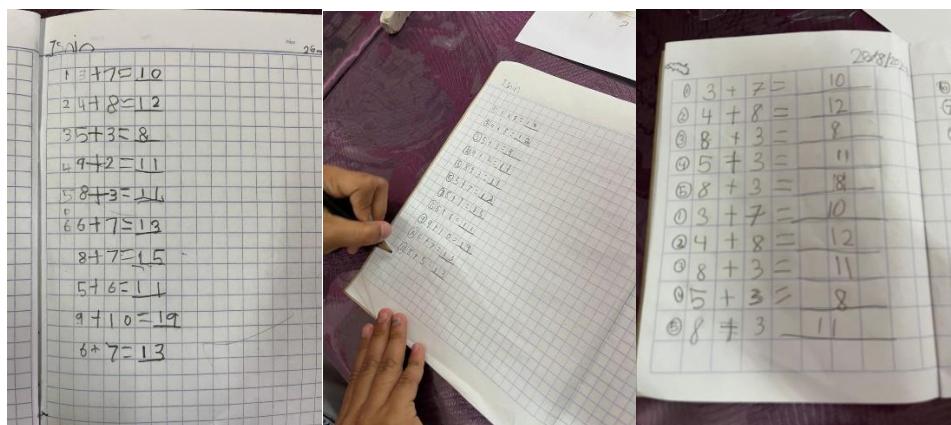


Figure 6 : Remedial pupils are able to solve the single digit addition with regrouping and write the correct answer on their exercise books.

Theme 2: Increased Engagement

Observation data shows that the pupils were noticeably more engaged and enthusiastic during sessions using the Laurnal frame compared to traditional methods (see Fig. 7). They were observed to be more focused and eager to participate in activities. This increased engagement was reflected in their interactions and enthusiasm for the learning process. The interview transcripts also supported the observation data. Noy shared: “The pupils said to me, ‘Teacher, when we have another mathematics session, we want to play with the blocks again.’”

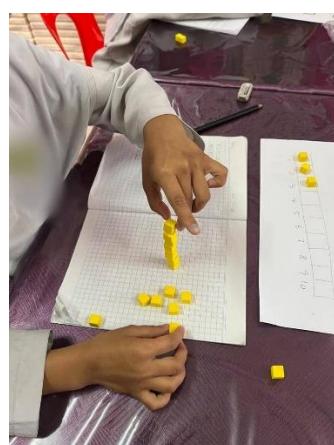


Figure 7 : Remedial pupil is engaged with the teaching aids and urged to manipulate the build the blocks

In the interview, Noy illustrated the impact of using the Laurnal's frame on the pupils' learning experience: "I noticed that when we used the Laurnal's frame, the students were not just passively listening—they were actively involved. They were eager to manipulate the blocks, try different combinations, and solve the addition problems themselves. This hands-on approach made them more excited about learning and improved their understanding of regrouping within 20. I could see that their confidence grew with each session, and they were more willing to take on challenging problems."

DISCUSSION & CONCLUSION

This study provides a significant window into how a remedial teacher help her remedial pupils to make sense of single digit addition with regrouping within 20 at remedial class and the effect of teaching and learning.

The Laurnal frame's design allows for a multi-modal approach to teaching single-digit addition. By physically manipulating counters, pupils engage in the enactive mode, gaining a hands-on experience of the addition process. This tactile interaction helps reinforce their understanding of regrouping. The iconic mode is employed as pupils visualize the number relationships within the frame, and the symbolic mode connects these visual and tactile experiences to numerical symbols and addition sentences. This comprehensive approach aligns with Bruner's theory of representation (1966) and supports the development of a deeper conceptual understanding of addition. The observation data and reflective journals indicate that pupils were able to grasp the concept of regrouping within 20 more effectively through the Laurnal's frame than through traditional methods (Laux, 2022). The physical and visual elements of the frame helped students to see and understand the process of carrying over numbers, which is a critical aspect of addition.

The study found that pupils were significantly more engaged and enthusiastic during sessions using the Laurnal's frame. This increased engagement can be attributed to the interactive and hands-on nature of the Laurnal's frame, which made learning more dynamic and enjoyable. The pupils' eagerness to participate and their positive feedback, as noted in the interviews, suggest that the Laurnal's frame effectively captures their interest and motivates them to actively participate in mathematical activities. Noy's observations and interviews reveal that the Laurnal's frame not only facilitated learning but also made the process enjoyable for the pupils. Their excitement about using the frame and their preference for it over traditional worksheets highlight the tool's success in enhancing motivation and engagement.

Additionally, the Laurnal frame's practical application in the classroom was evident through the students' ability to perform single-digit addition and regrouping tasks effectively. The step-by-step process demonstrated by Noy, including the use of visual aids and physical manipulation of blocks, allowed pupils to clearly see the results of their calculations. This approach helped students to internalize the concept of regrouping and apply it confidently in their addition problems. Results from the worksheets and observations confirm that pupils could accurately solve addition problems and understand the concept of regrouping within 20. The Laurnal's frame provided a clear visual representation of how regrouping affects the addition process, thus supporting the pupils' learning outcomes.

While the Laurnal frame has proven to be a valuable teaching tool, there are some limitations to consider. The study was conducted over a relatively short period, and further research is needed to assess the long-term impact of the Laurnal's frame on students' mathematical abilities.

Additionally, the effectiveness of the Laurnal's frame may vary based on individual learning styles and needs, and future studies could explore how to adapt the frame to better accommodate diverse learners. Future research could also investigate the Laurnal frame's applicability in other mathematical concepts beyond single-digit addition and its effectiveness across different age groups and educational settings. Exploring these areas could provide a more comprehensive understanding of the Laurnal frame's potential as a versatile teaching tool.

This study's main contribution was introducing a potentially practical mechanism, the Laurnal frame represents an innovative approach to teaching single-digit addition, effectively combining tactile, visual, and symbolic modes of learning. Its use in the classroom has shown to enhance pupils' conceptual understanding, increase engagement, and improve learning outcomes. By addressing both the practical and motivational aspects of teaching, the Laurnal's frame holds promise as a valuable educational tool for teaching mathematics. Further research and application will continue to refine its effectiveness and explore its potential in broader educational contexts.

REFERENCES

Ahdhianto, E., Marsigit, M., Haryanto, H., & Nurfauzi, Y. (2020). Improving fifth-grade pupils' mathematical problem-solving and critical thinking skills using problem-based learning. *Universal Journal of Educational Research*, 8(5), 2012-2021. <https://doi.org/10.13189/ujer.2020.080539>

Bartolini, M. G., & Martignone, F. (2020). Manipulatives in mathematics education. *Encyclopedia of Mathematics Education*, 487-494. https://doi.org/10.1007/978-3-030-15789-0_93

Bruner, J. S. (1966). *Toward a Theory of Instruction*. Harvard University Press.

Bryant, D. P., Pfannenstiel, K. H., Bryant, B. R., Roberts, G., Fall, A. M., Nozari, M., & Lee, J. (2021). Improving the mathematics performance of second-grade pupils with mathematics difficulties through an early numeracy intervention. *Behavior Modification*, 45(1), 99–121. <https://doi.org/10.1177/0145445519873651>

Carpenter, T. & Moser, J. (1984). The acquisition of addition and subtraction concepts in grades 1-3.

Clarke, V., & Braun, V. (2013). Teaching thematic analysis: Overcoming challenges and developing strategies for effective learning. *The Psychologists*, 62 (2), 120–123. <http://eprints.uwe.ac.uk/21155/3/Teaching%20thematic%20analysis%20Research%20Repository%20version.pdf>

Clerkin, A., & Gilligan, K. (2018). Pre-school numeracy play as a predictor of children's attitudes towards mathematics at age 10. *Journal of Early Childhood Research*, 16(3), 319–334. <https://doi.org/10.1177/1476718X18762238>

Clerkin, A., & Gilligan, K. (2018). Pre-school numeracy play as a predictor of children's attitudes towards mathematics at age 10. *Journal of Early Childhood Research*, 16(3), 319-334. <https://doi.org/10.1177/1476718X18762238>

Darling-Hammond, L., Flook, L., Cook-Harvey, C., Barron, B., & Osher, D. (2020). Implications for educational practice of the science of learning and development. *Applied Developmental Science*, 24(2), 97–140. <https://doi.org/10.1080/10888691.2018.1537791>

Darling-Hammond, L., Flook, L., Cook-Harvey, C., Barron, B., & Osher, D. (2019). Implications for educational practice of the science of learning and development. *Applied Developmental Science*, 24(2), 97-140. <https://doi.org/10.1080/10888691.2018.1537791>

Fatwana, H., Dasari, D., & Juandi, D. (2023). The use of learning aids for dyscalculia: Systematic literature review. *AL-ISHLAH: Jurnal Pendidikan*, 15(1), 639-652. <https://doi.org/10.35445/alishlah.v15i1.2633>

Fuson, K. C. (1992). Research on learning and teaching addition and subtraction of whole numbers. In G. Leinhardt, R. T. Putnam, & R. A. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 53–187). Lawrence Erlbaum Associates, Inc.

Fuson, K. C., & Briars, D. J. (1990). Using a base-ten blocks learning/teaching approach for first- and second-grade place-value and multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 21(3), 180–206. <https://doi.org/10.2307/749373>

Haylock, D., & Manning, R. (2019). *Mathematics explained for primary teachers*. SAGE.

Hinton, V., Flores, M., & Woods-Groves, S. (2024). The concrete-representational-Abstract integrated sequence as supplemental instruction to teach addition. *Journal of Education for Students Placed at Risk (JESPAR)*, 1-19. <https://doi.org/10.1080/10824669.2024.2321458>

Jones, J. P., & Tiller, M. (2017). Using Concrete Manipulatives in Mathematical Instruction. *Dimensions of Early Childhood*, 45(1), 18–23.

Kwon, H., & Capraro, M. M. (2021). Nurturing problem posing in young children: Using multiple representation within students' real-world interest. *International Electronic Journal of Mathematics Education*, 16(3), em0648. <https://doi.org/10.29333/iejme/11066>

Laux, K. (2022). An upper elementary teacher learns about making ten in first grade: Making a new ten, first grade (6-and 7-year-old) pupils. In *Educators' Learning from Lesson Study* (pp. 107-117). Routledge.

O'Leary, Z. (2014). *The essential guide to doing your research project* (2nd ed.). SAGE Publications one through three. *Journal for Research in Mathematics Education*, 15(3), 179–202. doi:10.2307/748348

Ozdem-Yilmaz, Y., Bilican, K. (2020). Discovery Learning—Jerome Bruner. In: Akpan, B., Kennedy, T.J. (eds) *Science Education in Theory and Practice*. Springer Texts in Education. Springer, Cham. https://doi.org/10.1007/978-3-030-43620-9_13

Raghbar, K. P., & Barnes, M. A. (2017). Early numeracy skills in preschool-aged children: a review of neurocognitive findings and implications for assessment and intervention. *The Clinical Neuropsychologist*, 31(2), 329–351. <https://doi.org/10.1080/13854046.2016.1259387>

Raghbar, K. P., & Barnes, M. A. (2017). Early numeracy skills in preschool-aged children: a review of neurocognitive findings and implications for assessment and intervention. *The Clinical Neuropsychologist*, 31(2), 329–351. <https://doi.org/10.1080/13854046.2016.1259387>

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.

Sunde, P. B., De Smedt, B., Verschaffel, L., & Sunde, P. (2023). Grade one single-digit addition strategies as predictors of grade four achievement in mathematics. *European Journal of Psychology of Education*. <https://doi.org/10.1007/s10212-023-00761-x>

Yin, R. K. (2018). *Case study research: design and methods* (6th ed.). Sage Publication.